Dynamic Constraint Solving

Handling Change and Uncertainty in Constraint Satisfaction

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1. Setting and requirements;

2. Reactive methods:
   (a) solution reuse;
   (b) reasoning reuse;

3. Proactive methods:
   (a) robust solutions;
   (b) flexible solutions;

4. Research directions.
1. **Setting** and **requirements**

2. **Reactive** methods:
   (a) **solution** reuse;
   (b) **reasoning** reuse;

3. **Proactive** methods:
   (a) **robust** solutions;
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4. Research directions.
Constraint solving in a dynamic setting
A dynamic world

Real world

- User
- Physical environment
- Other entities
- Physical system

Time
A dynamic reasoning system

Real world

User

Physical environment

Other entities

Physical system

Reasoning system

Time

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A reasoning system supervisor

![Diagram of reasoning system supervisor]

- Real world
  - User
  - Physical environment
  - Other entities
  - Physical system

- Reasoning system
  - Supervisor
    - Reasoning control

Time
A dynamic problem definition

- Real world:
  - User
  - Other entities
  - Physical environment
  - Physical system

- Supervisor:
  - Reasoning control

- Online problem definition:
  - Problem definition

- Reasoning system

- Time
A dynamic problem solving

- Real world
  - User
  - Other entities
  - Physical environment
  - Physical system

- Reasoning system
  - Supervisor
    - Reasoning control
  - Online problem definition
    - Problem definition
  - Online problem solving
    - Search control
    - Search algorithms

Time
A dynamic problem solution

Real world

User

Physical environment

Other entities

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Reasoning system

Supervisor

Reasoning control

Online problem definition

Online problem solving

Online problem solution

Problem definition

Search control

Search algorithms

Problem solution

Time

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An executive

Real world
- User
- Physical environment
- Other entities
- Physical system

Reasoning system

Supervisor
- Reasoning control

Executive
- Solution use

Online problem definition
- Problem definition

Online problem solving
- Search control
- Search algorithms

Online problem solution
- Problem solution

Time
A static problem processing

Real world
- User
- Physical environment
- Other entities
- Physical system

Reasoning system

Supervisor
- Reasoning control

Executive
- Solution use

Off line problem solving
- Problem pre-processing

Online problem definition
- Problem definition

Online problem solving
- Search control
- Search algorithms

Online problem solution
- Problem solution

Time

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Reactive and deliberative components

Reasoning system

 Supervisor

 Executive

 Real world

 User

 Physical environment

 Other entities

 Physical system

 Reactive control

 Deliberative control

 Off line problem solving

 Problem pre-processing

 Online problem definition

 Problem definition

 Online problem solving

 Search control

 Search algorithms

 Online problem solution

 Problem solution

 Time

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Main applications with change origins

1. online planning or scheduling → environment and physical system states, action effects, user objectives;
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5. **interactive** or **distributed** problem solving → user **choices**, other entities **choices**;
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6. **interactive** problem **specification**; constraint program **debugging**
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   → user **choices**, other entities **choices**;

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   → user **requirements**, problem **statement**;

7. **unordered search**
   → variable **assignments** or **unassignments**.
A reference application

A tool for **travel management**, embedded in a car, dedicated to the management of:

- the **route**;
- the tank **refueling** and car **maintenance**;
- the restaurant, hotel, and visit **reservations**;
- the **rendez-vous** with other people.

[Diagram of Travel Management Module with inputs: User requests, Vehicle position and state, Traffic and weather information, Other vehicles TMM, and outputs: Travel plan proposal(s).]
The main requirements in a dynamic setting

1. to minimize the **number of successive problem solvings** → to produce **robust** solutions i.e., solutions which **resist changes** as much as possible;
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4. to keep producing **consistent** and **optimal** solutions.
**A more formal definition**

**Nature** of the changes: any component of the problem definition:

- set of **variables**: additions or removals;
- **domain** definitions: extension or restriction;
- set of **constraints**: additions or removals;
- **constraint** variables: additions or removals;
- **constraint** definitions: extension or restriction.

Basic components: *Constraint addition* or *removal*
Do not confuse with

- **Conditional** or **composite** CSPs, whose solutions do not have all the same structure: this structure depends on choices for some variables → changes from a structure to another one are included in the problem definition;

- **Open** or **interactive** CSPs, whose variable values can be acquired during problem solving → variable domains are defined at solver’s request;

- CSPs that allow **dynamic systems** to be modelled, for example when representing planning/scheduling or situation tracking problems → system dynamics is included in a static problem definition.

With the problems we consider, changes come from outside the problem definition.
Two main classes of methods

- **Reactive** methods:
  - to use *no model* of the future changes;
  - when a change occurs, to *avoid* as much as possible *reasoning* again *from scratch*;

- **Proactive** methods:
  - to use *a model* of the future changes;
  - to be as *robust* as possible *facing* the possible *changes*.
1. **Setting** and **requirements**;

2. **Reactive** methods \( \Leftrightarrow \)
   
   (a) **solution** reuse;
   (b) **reasoning** reuse;

3. **Proactive** methods:
   
   (a) **robust** solutions;
   (b) **flexible** solutions;

4. **Research directions**.
Reactive methods

Main idea: to reuse as much as possible previous reasonings.

What is produced when reasoning on a problem, either from constraint propagation or from search? Information about:

1. problem consistency or inconsistency;

2. consistency or inconsistency of complete assignments;

3. global consistency or inconsistency of partial assignments;

4. local consistency or inconsistency of partial assignments.
To keep in mind

- **consistency** information is preserved by constraint **removals**: what was consistent remains consistent;

- **inconsistency** information is preserved by constraint **additions**: what was inconsistent remains inconsistent.

If a constraint removal is followed by a constraint addition, or if both occur in parallel, everything is **lost**, if nothing is done to **preserve information**.
Two classes of reactive methods

- **solution reuse**: to use *previous solutions* (consistency information), either as heuristic, or as search starting point

  → implicit assumption: a solution for the current problem exists *not too far* from the solutions which have been found for the previous problems, because problems are *close* to each other

  → using previous solutions tends to favour *efficiency* and solution *stability*;

- **reasoning reuse**: to reuse, among the *previously computed nogoods* (globally inconsistent partial assignments, inconsistency information), those that with certainty remain *valid*

  → to *save* as much as possible the results of the previous work: constraint propagation, search

  → can be seen as a form of *learning*. 
Solution reuse

1. **solution perturbation** → synthesis of a **sequence of methods** which allows consistency to be straightforwardly recovered from a previous solution; limited to **functional constraints**;

2. **tree search**: depth first search, limited discrepancy search . . . → use of the previous solution as a **heuristic**;

3. **local search**: heuristic repair, tabu search . . . → use of the previous solution as a **starting point**;

4. **variable unassignments** and **reassignments**: local changes → search for a **sequence of variable unassignments** and **reassignments** which allows consistency to be recovered from a previous solution.
An example with a graph colouring problem and the LC algorithm

<table>
<thead>
<tr>
<th>Variable</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{B,G}</td>
</tr>
<tr>
<td>2</td>
<td>{B,G}</td>
</tr>
<tr>
<td>3</td>
<td>{B,R}</td>
</tr>
<tr>
<td>4</td>
<td>{B,R}</td>
</tr>
</tbody>
</table>

Domain → \{B,R\}
Variable → 3

1 → 2
1 → 4

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A solution

Assigned value $\rightarrow$ B

Domain $\rightarrow$ \{B, R\}

1
{B, G}
G

2
{B, G}
B

3

4
{B, R}
R
A new constraint between variables 2 and 3

1: {B,G}
   G

2: {B,G}
   B

3: Tentative value → B
   Domain → {B,R}
   New unsatisfied constraint

4: {B,R}
   R

Tentative value → B
Domain → {B,R}
New unsatisfied constraint
Let us choose variable 2 and assign it value B.
Let us assign variable 3 value R
Let us come back to variable 2 and assign it value G
Let us assign variable 1 value B.

Tentative value → B
Domain → {B,R}

Assigned variable

Domain

Assigned value

1
{B,G}
B
2
{B,G}
3
4
{B,R}

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A new solution
Reasoning reuse

Difficulty: constraint removal.

A two-phase process, in case of constraint removal:

1. to remove only nogoods which became questionable;

2. to propagate constraints again from the nogoods which remain after phase 1.

Challenge: to limit as much as possible nogood removals in phase 1.
Two classes of reasoning reuse techniques

• **graph-based** techniques:
  → to use only **constraint graph** information to decide about the nogoods which became questionable;

• **justification-based** techniques:
  → to use **justifications** which have been computed and recorded during the propagation phase to decide about the nogoods which became questionable:
  
  – **minimal** justifications: for each nogood, recording of the constraint or set of constraints whose checking produced it;
  – **complete** justifications (**explanations**): for each nogood, recording of a set of constraints which implies it.
An example with a graph colouring problem and with complete justifications

Variable → Domain →

1 {R}
2 {B,R}
3 {B,G}
4 {B,R}
5 {B,G,R}
6 {G}
7 {B,G}
8 {G}
Arc-consistency enforcing on variable 2

1. \{R\}  removed value \{C12,\{\}\}
   Justification

2. \{B,R\}  removed value \{C12,\{\}\}

3. \{B,G\}

4. \{B,R\}

5. \{B,G,R\}

6. \{G\}

7. \{B,G\}

8. \{G\}
Arc-consistency enforcing on variable 3

1 \{R\}

2 \{B,R\}

3 \{B,G\}

4 \{B,R\}

5 \{B,G,R\}

6 \{G\}

7 \{B,G\}

8 \{G\}

{C12,\{\}}

{C23,\{R\}}
Arc-consistency enforcing on variable 4
Arc-consistency enforcing on variable 5
Arc-consistency enforcing on variable 7

1 \{R\}

2 \{B,R\}

3 \{B,G\}

4 \{B,R\}

5 \{B,G,R\}

6 \{G\}

7 \{B,G\}

8 \{G\}

\{C12,\{}\}

\{C23,\{R\}\}

\{C67,\{\}\}\n
\{C24,\{R\}\}

\{C35,\{B\}\}

\{C45,\{B\}\}
Arc-consistency enforcing on variable 5
Removal of constraint 12
Phase 1: updating of variable 2
Phase 1: updating of variable 3

Diagram:

- Node 1: {R}
- Node 2: {B,R}
- Node 3: {B,G}
- Node 4: {B,R}
- Node 5: {B,G,R}
- Node 6: {G}
- Node 7: {B,G}
- Node 8: {G}

Constraints:

- C24: {R}
- C35: {B}
- C45: {B}
- C57: {G}
- C67: {}

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Phase 1: updating of variable 4
Phase 1: updating of variable 5
Phase 2: Arc-consistency enforcing on variable 5
Phase 2: Arc-consistency enforcing on variable 4
Phase 2: Arc-consistency enforcing on variable 2

1
{R}

2
{B,R}

3
{B,G}

4
{B,R}

5
{B,G,R}

6
{G}

7
{B,G}

8
{G}

{C24,{R}}

{C45,{B,G}}

{C57,{G}}

{C58,{} }

{C67,{} }
To keep in mind

Reactive methods that reuse previous solutions or nogoods have the same worst case complexity as their static counterparts.

On some instances, reusing previous solutions or nogoods may be less efficient than solving the new problem from scratch.
1. **Setting** and **requirements**;

2. **Reactive** methods:
   (a) **solution** reuse;
   (b) **reasoning** reuse;

3. **Proactive** methods ⇐
   (a) **robust** solutions;
   (b) **flexible** solutions;

4. **Research directions**.
Two classes of proactive methods

To produce:

- **robust** solutions, which have every chance to remain solutions in spite of the changes that may occur;
- **flexible** solutions, which can be easily adapted when facing a change.
Robust solutions

Given some model of the possible changes, to produce solutions which will resist as best as possible these changes:

1. producing **stable solutions**, by favouring values that have every chance to be involved in solutions in the real world;

2. **probabilistic CSP** modelling: **uncertain constraints**; to search for complete assignments whose likelihood to be solution in the real world is the highest;

3. **mixed CSP** modelling: **controllable** and **uncontrollable variables**; to search for complete assignments of the controllable variables which will be solutions whatever the assignment of the uncontrollable variables is.

4. **stochastic constraint programming**: **controllable** and **uncontrollable variables**; to search for complete assignments of the controllable variables whose likelihood to be solution is the highest, taking into account the possible assignments of the uncontrollable variables.
Flexible solutions

To focus on **adaptability** in case of change:

1. producing and recording **sets of solutions**, by using **cross-product** representations;

2. producing and recording **conditional solutions**, by using more sophisticated representations such as **binary decision diagrams** (BDD) or **automata**;

3. producing **super solutions** i.e., solutions which, in case of a limited change (change of value for a limited number of variables), can recover consistency by performing a limited change too;

4. producing **partial solutions**, letting another process (for example, execution in planning and scheduling applications) making the remaining decisions according to the real situation: **least commitment** strategy.
1. **Setting** and **requirements**;

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4. **Research directions** ⇐
The study of the dynamic constraint solving setting as a whole, with all its requirements (robustness, flexibility, solution stability, optimality, efficiency ...), remains certainly to be done.
Need for revisiting constraint reasoning methods and constraint programming tools in order to make them able to deal with such dynamic settings:

→ able to handle efficiently two basic operations:
  ● constraint addition;
  ● constraint removal;

→ able to handle three types of variables:
  ● decision variables: controllable;
  ● random variables: uncontrollable;
  ● state variables: functions of the values taken by the variables of the previous two types.

→ able to reason at once about the dynamics of the world and about its own dynamics.
Auto-reasoning capabilities

Physical world

Reasoning system

Current state

Expected state

Reasoning system inputs
To make the connection with other frameworks and approaches, such as:

- stochastic satisfiability;
- bayesian networks;
- influence diagrams;
- Markov decision processes;
- game theory;
- temporal networks with uncertainty;
- anytime reasoning;
- time and resource-bounded computation.
More information

See [www.emn.fr/jussien/CP03tutorial](http://www.emn.fr/jussien/CP03tutorial)

- this presentation;
- a commented bibliography.