Column Generation and Constraint Programming for solving an Inventory Routing Problem in the Reverse Logistics Context

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1 Introduction

More and more manufacturers and distribution firms are confronted to the problem of "Reverse Logistics". Many different elements can be returned from their point of utilization to their point of origin (warehouse, plant,. . .): products having reached their life end, products to be repaired, packaging, spare parts, pallets. . . We consider a distribution system where products are stored at a central warehouse and a number of stores. There are three different flows: direct flow of products (from the warehouse to the stores), the reverse flows of materials (from the stores to the warehouse) and the internal flows of products (from stores to stores). In addition, we have to take into account the inventory problem. In fact, the stores cannot have a storage of products under their level of safety, so we must define the routes on all the periods of planification respecting this condition. Due to its complexity we decided to solve our problem thanks to hybrid methods (combining constraint programming and operations research techniques). Indeed, the expressiveness of constraint programming makes them a promising solution technique to solve our problem. In addition, column generation techniques are appropriate to the resolution of large problems. We will consequently solve our problem with column generation where subproblems will be solved by constraint programming techniques. Our problem is linked to the Vehicle Routing Problem (VRP) and particularly to the Inventory Routing Problem (IRP). In this kind of problem the routing and the inventory problems are solved together. Christiansen and Nygreen [2] solved an inventory routing problem with column generation. Our work differs here because of the mixed nature of the flows and the use of constraint programming to solve the subproblems in the column generation technique. One subproblem is used to generate feasible routes (as in the works of Rousseau *et al.* [4]) and another is used to plan feasible sequences of visiting days.

2 The master problem

Firstly, we define two concepts that will be used after : a *route* is a feasible order of visiting sites with the quantity of products collected, transferred and picked-up which respects the capacity of the vehicle and the time windows of the sites ; a *sequence* is a planning of visiting days for one site which respects the security level of storage and the capacity of the site. In this work we consider two subproblems: one for the routes determination and one for the inventory control with the sequence of visiting days, consequently we have two sets of binary variables. In fact, x_r is equal to 1 if route r is used in the optimal solution, 0 otherwise and θ_{is} is equal to 1 if sequence s is used for store i in the optimal solution, 0 otherwise. The objective is to minimize the routing and storage costs. Each route r has a cost c_r and each sequence s for the store i has a cost c_{is}. Different quantities that will be delivered, picked-up or transferred are determined with the resolution of the two subproblems. We have DS_{ist} and DR_{ir} which respectively represent the quantity of products delivered to store i on day t by sequence s and the quantity of products delivered to store i by route r. In the same way, we have PS_{ist} and PR_{ir} for the quantity of material picked-up and TS_{ist} and TR_{ir} for the products transferred between stores. Binary constants a_{3ir}, $a_{(3i+1)r}$ and $a_{(3i+2)r}$ are equal to 1 if respectively route r delivers some products to store i, collects materials from store i or transfers some products from store i. Binary constants b_{3ts}, $b_{(3t+1)s}$ and $b_{(3t+2)s}$ are equal to 1 if respectively sequence s delivers some products on day t, collects materials on day t or transfers some products on day t. Finally, k_{tr} is equal to 1 if route r is done on day t. We can write our model like this:

$$\text{Min : } z = \sum_r c_r x_r \quad + \quad \sum_i \sum_s c_{is}\theta_{is} \tag{1}$$

Subject to :

$$\forall i \quad \sum_s \theta_{is} \quad = \quad 1 \tag{2}$$

$$\forall t \quad \sum_r k_{tr} x_r \quad \leq \quad V \tag{3}$$

$$\forall i \forall t \quad \sum_r DR_{ir} a_{3ir} k_{tr} x_r \quad - \quad \sum_s DS_{is} b_{3ts} \theta_{is} = 0 \tag{4}$$

$$\forall i \forall t \quad \sum_r PR_{ir} a_{(3i+1)r} k_{tr} x_r \quad - \quad \sum_s PS_{is} b_{(3t+1)s} \theta_{is} = 0 \tag{5}$$

$$\forall i \forall t \quad \sum_r TR_{ir} a_{(3i+2)r} k_{tr} x_r \quad - \quad \sum_s TS_{is} b_{(3t+2)s} \theta_{is} = 0 \tag{6}$$

The first constraint represents the fact that one store has only one sequence. The second ensures that at most V vehicles are used. And finally, the three following constraints express the fact that the quantities determined in the route must be in coherence with these determined in the sequence. More details on the framework and the models of our research can be found in Grellier *et al.* [3], we will now give more informations on the first subproblem: the search of feasible routes.

3 The first stage of the resolution

Our work can be divided in several steps: **Firstly**, the resolution of the VRPPDTW (Vehicle Routing Problem with Pick-up and Delivery and Time Windows) in our network with only one warehouse with constraint programming techniques in order to use this in the search of feasible routes. **Then**, we will solve the same problem with column generation techniques. **Finally** we will solve the multi-periodic pick-up and delivery problem in a network with several warehouses. We solve the VRPPDTW with constraint programming techniques in order to used this for searching feasible routes with a negative reduced cost. These routes must be elementary paths (sub-tour elimination constraint (using the works of Rousseau et al. [4])) which respect: the time windows of the stores, the capacity of the vehicle (using the global constraint *cumulatives* of Beldiceanu and Carlsson [1]), the coherence between the quantity delivered to the stores and the capacity of delivery of the vehicle, . . . We use the Choco solver (choco.sf.net). Experiments and validation is being conducted on the instances of Breedam for the VRPPDTW (http://neo.lcc.uma.es/radi-aeb/WebVRP/). First results will presented during the meeting.

We address an inventory routing problem in mixed flows and propose hybrid methods to solve it: we use the column generation technique to solve our model, where subproblems are solved with constraint programming techniques. There are two subproblems: the route generation and the sequence generation. Preliminary results of the first stage of the resolution will presented at the meeting.

References

[1] N. Beldiceanu and M. Carlsson. "A New Multi-Resource *cumulatives* Constraint with Negative Heights". CP'2002, LNCS 2470, pp. 63-79.

[2] M. Christiansen and B. Nygreen. "A method for solving ship routing problems with inventory constraints". *Annals of Operations Research*, 81:357-378, (1998).

[3] E. Grellier, P. Dejax, N. Jussien and Zhiqiang Lu. "Vehicle routing problem in mixed flows for reverse logistics: a modeling framework". ILS 2006.

[4] L.M. Rousseau, M. Gendreau, G. Pesant. "Solving small VRPTWs with Constraint Programming Based Column Generation". CP-AI-OR'02. (2002)