ECOLE DES MINES DE NANTES

Improving branch-and-bound exploration for Open-Shop problems

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Outline

- Open Shop problems
- A branch-and-bound exploration for OS
- What's wrong with chronological backtracking
- Improving tree searches
- First results
- Future works

The Open Shop problem

- Data : *n* jobs composed of *m* tasks to process through *m* machines
- Constraints
  - a machine can process only one task at a time
  - two tasks of a same job cannot overlap
- Criterion : smallest makespan
- Complexity : NP-hard for $m \geq 3$

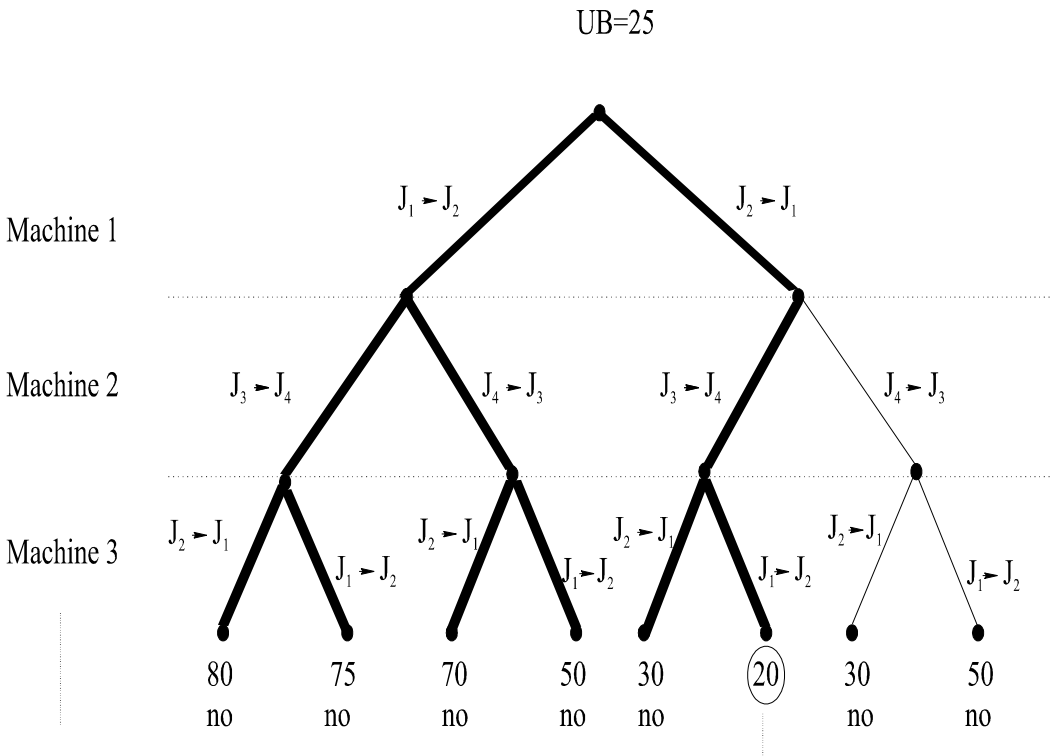
Solving Open Shop problems

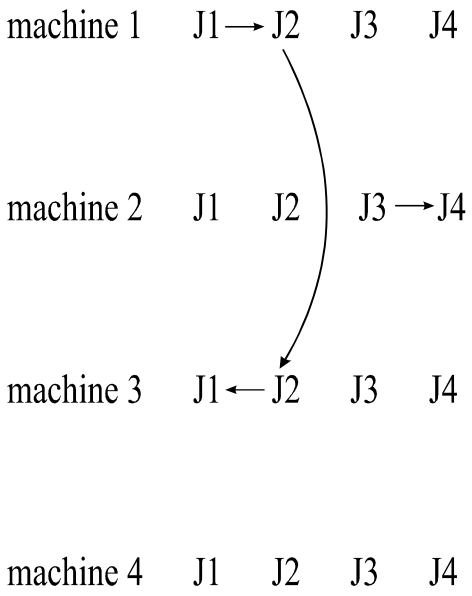
- Good heuristics have been developed
  - *Gueret & Prins 98*
- A few branch and bound exploration
  - the best one : *Brucker et al.* ??
  - Some 7x7 problems are still unsolved

Brucker *et al.* B&B

- Depth-first search algorithm
- Node creation
  - build a heuristic solution
  - compute a critical path on that solution
  - fix disjunctions on that critical path
    - branching scheme of *Grabowski et al.*
- Node propagation
  - disjunctions are propagated using *immediate selections* from *Carlier & Pinson*

Drawbacks of DFS





Drawbacks of DFS

- Unnecessary explorations

- *Thrashing*

  – doing over and over the same unusefull explorations

Overcoming the drawbacks

- Basic principle
  - recording **information** during the search
  - using that information to compute a **relevant** backtrack point when encountering a contradiction
  - using that information to determine **reusable** parts of previous computations
- Basic tool
  - **explanation** : set of nodes that leads to the attached action
    - modification of head or tail of a task
    - inferred precedence constraint

Using explanations

- Backtrack occurs when rejecting a node
- What can be done
  - there is no valid start date for a given task
    - Explanation E : union of the explanations of all modifications made on the associated variable
  - all the children of a given node have been tested
    - Explanation E : union of the explanations of each unsuccessful child
- The most recent **relevant** backtrack point is the most recent node in E

Intelligent Backtracking

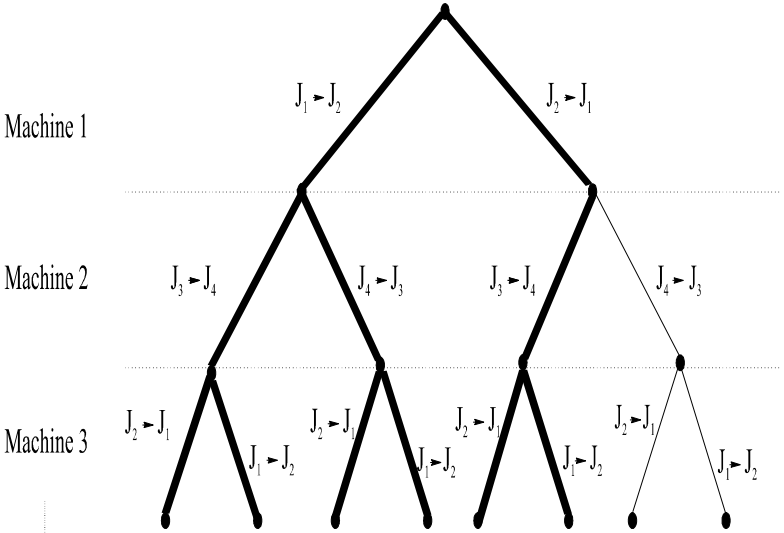
A more complete usage

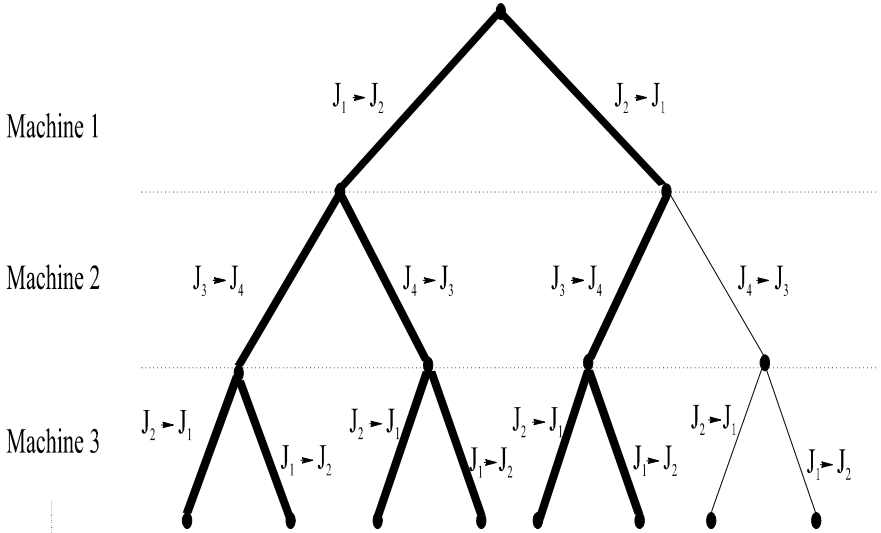
- Saving computation
  - Let *n* be the chosen node for backtracking
  - every explanation that **does not** contain *n* is still **valid**
  - backtracking is replaced by **path modification**

- Remaining complete
  - some more information is stored

Dynamic Backtracking

Graphical comparison



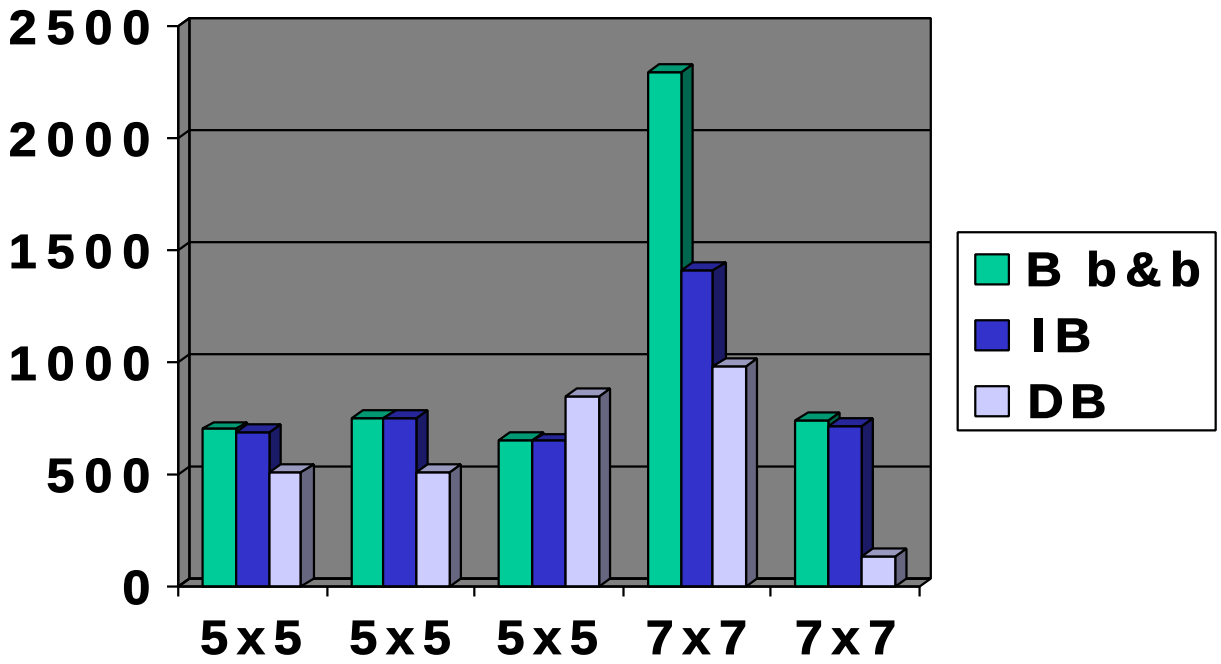


Complexity issues

- spatial complexity overhead: O(e.n.d)
  - n = number of variables + number of precedences
  - d = depth of the search tree
  - e = size of an explanation (O(d))
- time complexity overhead: polynomial
  - union of a polynomial number of explanations of polynomial size during constraint propagation

Computational results

- Tests performed on *Taillard's* problems



Computational results

- General observations
  - Intelligent Backtracking always improves the search up to 90% for some problems
  - Time overhead is widely compensated by the improvements in number of developed nodes
  - Dynamic Backtracking is more stable for series of similar problems : it depends less on the data
- Specific results
  - IB allowed to solve a 10x10 open problem
  - DB can greatly improve results but can also get caught in the *bad* part of the search space

Conclusion and Future works

- **Recording information** during the search may greatly improve obtained results

- Dynamic Backtracking is promising but too restricted by completeness constraints
  - currently testing an heuristic method based on that idea
    - taking rid of the completeness constraints
    - using heuristic techniques to determine the next modification
    - sort of tabu search on the paths