

Removing more values than Max-restricted path consistency for the same cost

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Abstract. Filtering techniques are essential to efficiently look for a solution in a constraint network (CN). They remove some local inconsistencies and so reduce the search space. However, a given local consistency has to be not too expensive if we want to use it to efficiently prune the search tree during search. Hence, for a long time it has been considered that the best choice is the limited local consistency achieved by forward checking [14, 16]. However, more recent works [17, 4, 15] show that maintaining arc consistency (which is a more pruningful local consistency) during search outperforms forward checking on hard and large constraint networks. It is very likely that maintaining an even more pruningful local consistency may pay off on very hard problems. A comparison of the local consistencies more pruningful than arc consistency that can be used on large CNs has been done in [8]. The conclusion of this work is that Max-restricted path consistency (Max-RPC, [7]) is one of the most promising local consistencies. It deletes far more values than arc consistency with reasonable cpu time requirements. For all the local consistencies more pruningful than Max-RPC, there exists CNs on which achieving them is really prohibitive. In this paper we show that Max-RPC is not the limit we have to do not exceed to guarantee a reasonable cpu time. We propose a new filtering algorithm, called Max-RPCen1, which prunes more values than an algorithm achieving Max-RPC. It requires less constraint checks than Max-RPC1, the Max-RPC algorithm proposed in [7], and has almost the same cpu time requirements.

1 Introduction

Finding a solution in a constraint network (CN) involves looking for an assignment of values for the problem variables so that all the constraints are simultaneously satisfied. This task is NP-hard, and to avoid a combinatorial explosion, the search space has to be reduced by filtering techniques, which remove some local inconsistencies. Obviously, a given local consistency can advantageously be maintained during search only if it requires less cpu time to detect that a branch of the search tree does not lead to any solution than a search algorithm

to explore this branch. For a long time, the only practicable local consistency was arc consistency (AC, namely 2-consistency or (1, 1)-consistency in the formalism of [10]). Indeed, higher levels of k -consistency, such as path consistency ($k=3$), are so expensive that they can be used only on very small CNs. In the last three years, new local consistencies have been proposed and a comparison of those that can be used on large CNs have been done in [8] considering both their pruning efficiency and the time required to achieve them. The conclusion of this comparison is that Max-RPC [7] is one of the most worthwhile local consistencies. This local consistency is far more pruningful than arc consistency and although Max-RPC1 (the Max-RPC algorithm proposed in [7]) has a worst case time complexity close to the one of the best path consistency algorithm, experiments in [7, 8] show that Max-RPC1 has good cpu time performances. These experiments show also that the cpu time required to enforce the local consistencies more pruningful than Max-RPC, such as neighborhood inverse consistency [12] and singleton consistencies [6], is not of the same order of magnitude. In this paper we show that Max-RPC is not the limit we have to do not exceed to guarantee a reasonable cpu time. We propose a new filtering algorithm called Max-RPCen1. A pruning efficiency study shows that it is significantly more pruningful than an algorithm achieving Max-RPC. Furthermore, experiments highlight that Max-RPCen1 requires less constraint checks than Max-RPC1, and is efficient in time.

2 Definitions and notations

A *network of binary constraints* $P = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ is defined by a set $\mathcal{X} = \{i, j, \dots\}$ of n variables, each taking value in its respective finite *domain* D_i, D_j, \dots elements of \mathcal{D} and a set \mathcal{C} of e binary constraints. d is the size of the largest domain. A *binary constraint* C_{ij} is a subset of the Cartesian product $D_i \times D_j$ that denotes the compatible pairs of values for i and j . We note $C_{ij}(a, b) = \text{true}$ to specify that $((i, a), (j, b)) \in C_{ij}$. We then say that (j, b) is a *support* for (i, a) on C_{ij} . Checking whether a pair of value is allowed by a constraint is called a *constraint check*. With each CN we associate a *constraint graph* in which nodes represent variables and arcs connect pairs of variables that are constrained explicitly. c is the number of 3-cliques in the constraint graph. The *neighborhood* of i is the set of variables linked to i in the constraint graph. An *instantiation* of a set of variables S is a set of value assignments $\{I_j\}_{j \in S}$, one for each variable belonging to S s.t. $\forall j \in S, I_j \in D_j$. An instantiation I of S satisfies a constraint C_{ij} if $\{i, j\} \not\subseteq S$ or $C_{ij}(I_i, I_j)$ is true. An instantiation is *consistent* if it satisfies all the constraints. A pair of values $((i, a), (j, b))$ is *path consistent* if for all $k \in \mathcal{X}$ s.t. $j \neq k \neq i \neq j$, this pair of values can be extended to a consistent instantiation of $\{i, j, k\}$. Checking whether a pair of values is path consistent is called a *path consistency check*. (j, b) is a *path consistent support* for (i, a) if $(a, b) \in C_{ij}$ and $((i, a), (j, b))$ is path consistent. A *solution* of $P = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ is a consistent instantiation of \mathcal{X} . A value (i, a) is *consistent* (or *completable*

[11]) if there is a solution I such that $I_i = a$, and a CN is *consistent* if it has at least one solution.

3 From arc consistency to conservative path consistency

Let us recall that a k -consistency algorithm removes the instantiations of length $k - 1$ that cannot be extended to a consistent instantiation including any additional k^{th} variable. So, an arc consistency algorithm ($k = 2$) deletes the values that have not at least one compatible value (a support) on each constraint. Higher levels of k -consistency, such as path consistency (PC, $k = 3$), are so expensive that they can be used only on very small CNs. A path consistency algorithm has to try to extend all the pairs of values, even those between two variables that are not linked by a constraint, to any third variable. Thus, even the most efficient PC algorithms [5, 18] are prohibitive. Obviously, enforcing an higher level of k -consistency is even more expensive. Furthermore, if $k > 2$ a k -consistency algorithm changes the structure of the network. Indeed, constraints involving $k - 1$ variables may have to be added to store the deletion of the $(k - 1)$ -inconsistent instantiations. To avoid these important drawbacks, a restricted path consistency algorithm (RPC, [1]) performs only the most pruningful path consistency checks, namely those that can directly lead to the deletion of a value, and it deletes only values in order to keep unchanged the structure of the network. In addition to AC, an RPC algorithm checks the path consistency of the pairs of values $((i, a), (j, b))$ such that (j, b) is the only support for (i, a) in D_j . If such a pair of values is path inconsistent, its deletion would lead to the arc inconsistency of (i, a) , and thus (i, a) can be removed. So, these few path consistency checks allow to remove more values than arc consistency while leaving unchanged the set of constraints. We can extend the idea of RPC to remove more values by checking the existence of a path consistent support on a constraint not only for the values that have only one support on this constraint as in RPC, but also for the values having at most k supports on this constraint (k -restricted path consistency, k -RPC [7]), or for all the values, whatever is the number of supports they have (Max-restricted path consistency). Considering the pruning efficiency, Max-RPC is an upper bound for k -RPC and a Max-RPC algorithm removes all the k -restricted path inconsistent values for all k . However, we can delete even more values than Max-RPC, still without adding any constraint in the network.

The limit in terms of pruning efficiency of checking the path consistency of pairs of values while keeping the structure of the network is conservative path consistency (CPC). CPC is the restriction of strong path consistency (a strong PC algorithm enforces both arc and path consistency) to the explicit constraints of the network. If there is no constraint between two variables i and j , any pair of values $((i, a), (j, b))$ is conservative path consistent. If i and j are linked by a constraint $C_{ij} \in \mathcal{C}$, a pair of values $((i, a), (j, b))$ allowed by this constraint is conservative path consistent if, and only if, for any third variable $k \in \mathcal{X}$ linked to both i and j , $((i, a), (j, b))$ can be extended to a consistent instantiation

- A binary CN is (i, j) -consistent iff $\forall i \in \mathcal{X}, D_i \neq \emptyset$ and any consistent instantiation of i variables can be extended to a consistent instantiation including any j additional variables.
- A domain D_i is arc consistent iff, $\forall a \in D_i, \forall j \in \mathcal{X}$ s.t. $C_{ij} \in \mathcal{C}$, there exists $b \in D_j$ s.t. $C_{ij}(a, b)$. A CN is *arc consistent* ((1, 1)-consistent) iff $\forall D_i \in \mathcal{D}, D_i \neq \emptyset$ and D_i is arc consistent.
- A pair of variables (i, j) is path consistent iff $\forall (a, b) \in C_{ij}, \forall k \in \mathcal{X}$, there exists $c \in D_k$ s.t. $C_{ik}(a, c)$ and $C_{jk}(b, c)$. A CN is *path consistent* ((2, 1)-consistent) iff $\forall i, j \in \mathcal{X}, (i, j)$ is path consistent.
- A binary CN is *strongly path consistent* iff it is node consistent, arc consistent and path consistent.
- A binary CN is *restricted path consistent* iff
 $\forall i \in \mathcal{X}, D_i$ is a non empty arc consistent domain and,
 $\forall (i, a) \in D, \forall j \in \mathcal{X}$ s.t. (i, a) has only one support b in D_j ,
for all $k \in \mathcal{X}$ linked to both i and j ,
 $\exists c \in D_k$ s.t. $C_{ik}(a, c) \wedge C_{jk}(b, c)$.
- A binary CN is *max restricted path consistent* iff
 $\forall i \in \mathcal{X}, D_i$ is a non empty arc consistent domain and,
 $\forall (i, a) \in D$, for all $j \in \mathcal{X}$ linked to i ,
 $\exists b \in D_j$ s.t. $C_{ij}(a, b)$ and for all $k \in \mathcal{X}$ linked to both i and j ,
 $\exists c \in D_k$ s.t. $C_{ik}(a, c) \wedge C_{jk}(b, c)$.
- A pair of values $((i, a), (j, b))$ such that there is no constraint $C_{ij} \in \mathcal{C}$ is conservative path consistent. If $\exists C_{ij} \in \mathcal{C}$, a pair of values $((i, a), (j, b))$ is conservative path consistent iff $C_{ij}(a, b)$ and $\forall k \in \mathcal{X}$ linked to both i and j , $\exists c \in D_k$ s.t. $C_{ik}(a, c) \wedge C_{jk}(b, c)$. A constraint $C_{ij} \in \mathcal{C}$ is conservative path consistent iff all the pairs of values $((i, a), (j, b))$ s.t. $C_{ij}(a, b)$ are conservative path consistent. A CN is *conservative path consistent* iff $\forall (i, a) \in \mathcal{D}, (i, a)$ is arc consistent and $\forall C_{ij} \in \mathcal{C}, C_{ij}$ is conservative path consistent.

Fig. 1: The mentioned local consistencies

including k . A constraint network is conservative path consistent iff it is arc consistent and all the pairs of values allowed by the constraints explicitly present in the network are conservative path consistent (see Fig. 1). We can remark that the deletion of an arc inconsistent value can make conservative path inconsistent a pair of values, and conversely, the deletion of a pair of values can make arc inconsistent some values. Fig. 2 shows an example. On this CN, all the pairs of values are conservative path consistent. However, enforcing AC removes $(2, c)$ and $(5, c)$, and these deletions leads to the conservative path inconsistency of all the pairs of values between $(2, b), (2, c), (3, a), (4, b), (5, a)$, and $(5, c)$. The deletion of these pairs of values, at their turn, makes arc inconsistent the values $(2, b)$ and $(5, a)$. So, a CPC algorithm cannot be composed of two independent steps, removing first the conservative path inconsistent pairs of values and then the arc inconsistent values (or in reverse order). Furthermore, the behavior of CPC is dependent on the structure of the constraint graph. If two variables i and j are not neighbors, we can add an universal constraint allowing all the pairs of values $(a, b) \in D_i \times D_j$ between i and j . The resulting CN is equivalent to the initial one since it has the same set of solutions. However, a CPC algorithm can delete more values if we add universal constraints between variables that are not

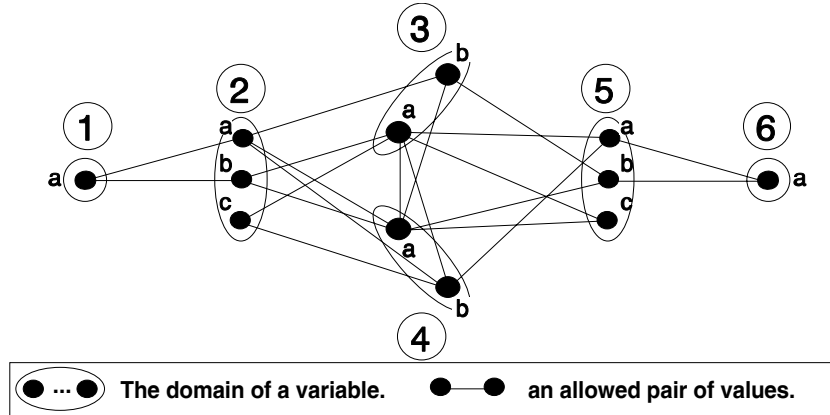


Fig. 2: A CN on which the deletion of arc inconsistent values makes conservative path inconsistent some pairs of values and conversely.

neighbors. Obviously, this process increases the time complexity. On complete constraint networks, CPC is equivalent to strong path consistency. Therefore, CPC is more pruningful than Max-RPC (see Section 5) and does not change the structure of the network, but on complete constraint networks it is as expensive as strong path consistency. Furthermore, in real applications, a constraint is seldom represented by a Boolean matrix or a set of compatible pairs of values. They are often represented by a predicate which has a particular semantics (\neq , \leq , ...). Enforcing CPC leads to the generation of the Boolean matrix to store the deletion of the conservative path inconsistent pairs of values and the semantics of the constraints is lost. The drawbacks of CPC are too important, and enforcing it is too expensive to be worthwhile. So, in the following, we study the pruning efficiency of CPC, but we do not try to know the cpu time required to achieve it.

To avoid the drawbacks of CPC while removing more values than an algorithm achieving Max-RPC, the new filtering algorithm proposed in the next section, called Max-RPCEn1, does not try to check the conservative path consistency of all the pairs of values. Max-RPCEn1 does not delete any pair of values and it performs more value deletions than an algorithm achieving Max-RPC only when the enforcement of Max-RPC allows it to detect some conservative path inconsistent values.

4 Max-RPCEn1

4.1 Bases of the algorithm

Max-RPCEn1 is an improvement on Max-RPC1. Max-RPC1 is based on the idea of AC6 [2] which is a very efficient arc consistency algorithm but that does not use the bidirectionality of the constraints, namely the property that a value (i, a) is a support of another value (j, b) if and only if (j, b) is a support for

(i, a) . Like AC7 [3], Max-RPCEn1 uses this property of the constraints to infer the existence, or the non-existence of supports. This substantially reduces the number of constraint checks and the cpu time required to enforce the local consistency on most of the CNs. Since all the constraints of a CN are bidirectional, if we find that a value (j, b) is a support for (i, a) , we can infer that (i, a) is a support for (j, b) . Furthermore, if we have found that (i, a) is not compatible with any value lower than b in D_j , it is useless to check whether (i, a) is compatible with (j, b') (with $b' < b$) when we look for a support for (j, b') in D_i . Taking advantage of the bidirectionality to achieve Max-RPC leads to even more savings than during the enforcement of AC. Indeed, (j, b) is a path consistent support for (i, a) if and only if (j, b) is a path consistent support for (i, a) . So, the bidirectionality of the constraints allows to infer some path consistent supports. This not only avoids some constraint checks, but also some path consistency checks.

The second improvement of Max-RPCEn1 on Max-RPC1 is an enhancement of the pruning efficiency. We say that a pair of values $((i, a), (j, b))$ is *valid path consistent* if for all the 3-cliques $\{i, j, k\}$ of the constraint graph there exists a value $c \in D_k$ compatible with both (i, a) and (j, b) such that Max-RPCEn1 has not found the conservative path inconsistency of $((i, a), (k, c))$ or $((j, b), (k, c))$. A value (j, b) is a valid path consistent support for (i, a) if (j, b) is compatible with (i, a) and $((i, a), (j, b))$ is valid path consistent. The values deleted by Max-RPCEn1 are those that do not have any valid path consistent support on a constraint. So, Max-RPCEn1 removes the Max-restricted path inconsistent values and some of the conservative path inconsistent values. The data structure that allows Max-RPCEn1 to detect the conservative path inconsistency of some pairs of values is the array of counters M . $M_{ija} = b$ if there is no valid path consistent support of (i, a) in D_j lower than b . To know whether a pair of value $((i, a), (j, b))$ allowed by a constraint C_{ij} is path consistent w.r.t. a third variable k linked to both i and j , we have to look for a value $c \in D_k$ that is compatible with (i, a) and (j, b) . However, even if such a value c exists, a CPC algorithm can delete $((i, a), (j, b))$ if $((i, a), (k, c))$ or $((j, b), (k, c))$ is conservative path inconsistent. If a support $c \in D_k$ of (i, a) is lower than M_{ika} or if $a < M_{ikc}$ then $((i, a), (k, c))$ is conservative path inconsistent. This is the property used by Max-RPCEn1 to detect some conservative path inconsistent pairs of values. Therefore, a pair of values $((i, a), (j, b))$ is valid path consistent if for all the 3-cliques $\{i, j, k\}$ of the constraint graph there exists a value $c \in D_k$ compatible with both (i, a) and (j, b) such that $(c \geq M_{ika}) \wedge (c \geq M_{jkb}) \wedge (a \geq M_{kic}) \wedge (b \geq M_{kjc})$.

4.2 The Algorithm

The data structures of Max-RPCEn1 are:

- each initial domain is considered as the integer range $1..|D_i|$. The current domain is represented by a table of booleans. We use the following constant time functions and procedures to handle the current domain:

```

procedure Max-RPCEn1();
1  DeletionList  $\leftarrow$   $\emptyset$ ; InitList  $\leftarrow$   $\emptyset$ ;
2  forall  $(i, a) \in \mathcal{D}$  do
3     $S_{ia}^{PC} \leftarrow \emptyset$ ;
4    forall  $C_{ij} \in \mathcal{C}$  do
5       $S_{ija} \leftarrow \emptyset$ ;  $M_{ija} \leftarrow nil$ ; InitList  $\leftarrow$  InitList  $\cup$   $\{(i, j), a\}$ ;
6  forall  $C_{ij} \in \mathcal{C}$  s.t.  $i < j$  do
7    Common[ $C_{ij}$ ]  $\leftarrow \emptyset$ ;
8    forall  $C_{jk} \in \mathcal{C}$  do
9      if  $\exists C_{ik} \in \mathcal{C}$  then Common[ $C_{ij}$ ]  $\leftarrow$  Common[ $C_{ij}$ ]  $\cup$   $\{k\}$ ;
10   Common[ $C_{ji}$ ]  $\leftarrow$  Common[ $C_{ij}$ ];
11  while InitList  $\neq \emptyset$  or DeletionList  $\neq \emptyset$  do
12    if DeletionList  $\neq \emptyset$  then
13      choose and delete  $(i, a)$  from DeletionList;
14      PropagDeletion( $i, a, DeletionList$ );
15    else
16      choose and delete  $[(i, j), a]$  from InitList;
17      if  $(i, a) \in D$  and not HasAValidPCSupport( $i, a, j$ ) then
18        remove( $D_i, a$ ); DeletionList  $\leftarrow$  DeletionList  $\cup$   $\{(i, a)\}$ ;

procedure PropagDeletion( $j, b$ , in out DeletionList);
1  while  $S_{jb} \neq \emptyset$  do
2    choose and delete  $(i, a)$  from  $S_{jb}$ ;
3    if  $a \in D_i$  and not HasAValidPCSupport( $i, a, j$ ) then
4      remove( $D_i, a$ ); DeletionList  $\leftarrow$  DeletionList  $\cup$   $\{(i, a)\}$ ;
5  while  $S_{jb}^{PC} \neq \emptyset$  do
6    choose and delete  $((i, a), (k, c))$  from  $S_{jb}^{PC}$ ;
7    if  $a \in D_i$  and  $c \in D_k$  and  $a \in S_{kic}$  then
8       $b' \leftarrow b$ ;
9      if IsValidPathConsistent( $i, a, k, c, j, b'$ ) then
10        $S_{jb'}^{PC} \leftarrow S_{jb'}^{PC} \cup \{((i, a), (k, c))\}$ ;
11     else
12       remove  $a$  from  $S_{kic}$ ;
13       if  $c \in S_{ika}$  then
14         remove  $c$  from  $S_{ika}$ ;
15         if not HasAValidPCSupport( $k, c, i$ ) then
16           remove( $D_k, c$ ); DeletionList  $\leftarrow$  DeletionList  $\cup$   $\{(k, c)\}$ ;
17         if not HasAValidPCSupport( $i, a, k$ ) then
18           remove( $D_i, a$ ); DeletionList  $\leftarrow$  DeletionList  $\cup$   $\{(i, a)\}$ ;

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Fig. 3: Max-RPCEn1.

- $next(D_i, nil)$ returns the smallest value of D_i if $D_i \neq \emptyset$ and ∞ otherwise.
 - $next(D_i, a)$ with $a \neq nil$ returns the smallest value in D_i greater than a if a is not the greatest value of D_i , and ∞ otherwise.
 - $remove(D_i, a)$ removes the value a from D_i .
- A value a is in S_{jib} if (i, a) is currently supported by (j, b) i.e. b is the current valid path consistent support of (i, a) on C_{ij} . In other words, S_{jib} is the set of the values that may no longer have a valid path consistent support in D_j if we delete (j, b) . S_{jib} is the set of the values (i, a) such that $a \in S_{jib}$ for some $C_{ij} \in \mathcal{C}$. If $S_{jib} \neq \emptyset$, $first(S_{jib})$ returns the first value in S_{jib} and ∞ otherwise.
- To take advantage of the bidirectionality of the constraints and to detect the conservative path inconsistency of some pairs of values, Max-RPCEn1 uses the array M . $M_{ija} = b$ if $\forall b' \in D_j$ s.t. $b' < b$, (j, b') is not a valid path consistent support for (i, a) , namely (j, b') is not compatible with (i, a) , $((i, a), (j, b'))$ is path inconsistent, or Max-RPCEn1 has found the conservative path inconsistency of $((i, a), (j, b'))$.

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function HasAValidPCSupport(i, a, j) : boolean;
1  while  $S_{ija} \neq \emptyset$  do
2     $b \leftarrow \text{first}(S_{ija})$ 
3    if  $b \notin D_j$  then remove  $b$  from  $S_{ija}$ 
4    else  $S_{jib} \leftarrow S_{jib} \cup \{a\}$ ; return true;
5   $M_{ija} \leftarrow \text{next}(D_j, M_{ija})$ ;
6  while  $M_{ija} \neq \infty$  do
7    if ( $a > M_{jiM_{ija}}$ ) and ( $C_{ij}(a, M_{ija})$ ) then
8       $ValidPC \leftarrow true$ ;
9      forall  $k \in Common[C_{ij}]$  while  $ValidPC$  do
10        $c \leftarrow nil$ ;
11       if IsValidPathConsistent(i, a, j, Mija, k, c) then  $CS[k] \leftarrow c$ ;
12       else  $ValidPC \leftarrow false$ ;
13     if  $ValidPC$  then
14        $S_{jiM_{ija}} \leftarrow S_{jiM_{ija}} \cup \{a\}$ ;
15       forall  $k \in Common[C_{ij}]$  do
16          $S_{kCS[k]}^{PC} \leftarrow S_{kCS[k]}^{PC} \cup \{(i, a), (j, M_{ija})\}$ ;
17       return true;
18      $M_{ija} \leftarrow \text{next}(D_j, M_{ija})$ ;
19 return false;

function IsValidPathConsistent(i, a, j, b, k, in out c) : boolean;
1  if ( $M_{ika} = M_{jkb}$ ) then
2    if  $M_{ika} \in D_k$  then
3       $c \leftarrow M_{ika}$ ; return true;
4    else  $c \leftarrow \max(c, M_{ika})$ ;
5    else  $c \leftarrow \max(c, M_{ika}, M_{jkb})$ ;
6    if  $c \notin D_k$  then  $c \leftarrow \text{next}(D_k, c)$ ;
7    while  $c \neq \infty$  do
8      if ( $a \geq M_{kic}$ )  $\wedge$  ( $b \geq M_{kjc}$ )  $\wedge$  ( $C_{ik}(a, c)$ )  $\wedge$  ( $C_{jk}(b, c)$ ) then return true;
9       $c \leftarrow \text{next}(D_k, c)$ ;
10 return false;

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Fig. 4: The functions used by Max-RPCEN1.

- If $((i, a), (j, b)) \in S_{kc}^{PC}$ and $(a \in D_i \wedge b \in D_j)$, b is the current valid path consistent support of (i, a) in D_j and (k, c) is currently supporting $((i, a), (j, b))$, i.e. (k, c) is such that $(C_{ik}(a, c) \wedge C_{jk}(b, c))$ and Max-RPCEN1 has not found that $((i, a), (k, c))$ or $((j, b), (k, c))$ are conservative path inconsistent. So, S_{kc}^{PC} is the set of the pairs of values that may be no longer valid path consistent w.r.t. k if we delete (k, c) .
- $Common[C_{ij}]$ is the set of the variables k that are linked to both i and j , i.e. the variables k such that $\{i, j, k\}$ is a 3-clique in the constraint graph.
- An arc-value pair $[(i, j), a]$ is in *InitList* if Max-RPCEN1 has not yet checked whether (i, a) has a valid path consistent support in D_j . A value (j, b) is in *DeletionList* if b has been removed from D_j but this deletion has not been propagated yet.

For each arc-value pair $[(i, j), a]$ Max-RPCEN1 (see Fig. 3) uses the function *HasAValid-PCSupport* (Fig. 4) to know whether (i, a) has a valid path consistent support on C_{ij} . This function tries first (lines 1 to 4) to infer a valid path consistent support looking for an undeleted value in S_{ija} , i.e. in the list of the values supported by (i, a) on C_{ij} . If no valid path consistent support can be inferred, Max-RPCEN1 goes on with its search looking for the smallest valid path consistent support in D_j . The array M allows to reduce the number of constraint checks performed. *HasAValidPCSupport* does not check (j, M_{ija}) (see line 5) because it is not a valid path consistent support for (i, a) . Indeed, if it is the first time that *HasAValidPCSupport* is called for the pair arc value

$[(i, j), a]$, $M_{ija} = nil$, otherwise *HasValidPCSupport* has been called by *PropagDeletion* (Fig. 3) because the current valid path consistent support of (i, a) on C_{ij} has been deleted and (j, M_{ija}) is no longer in D_j . Furthermore, by definition of M_{ija} , there is no valid path consistent support of (i, a) in D_j lower than M_{ija} . So, if (i, a) has a valid path consistent support in D_j , it is greater than M_{ija} . Furthermore, if $b \in D_j$ is a valid path consistent support for (i, a) , $a > M_{jib}$ since if $a < M_{jib}$ (i, a) is not a valid path consistent support for (j, b) and if $a = M_{jib}$, b would have been found in S_{ija} . *IsValidPathConsistent* (Fig. 4) is used to know whether a pair of values $((i, a), (j, b))$ is valid path consistent w.r.t. a third variable k . This function looks for the smallest value c in D_k supporting $((i, a), (j, b))$, namely such that $((i, a), (k, c))$ and $((j, b), (k, c))$ are allowed pairs of values not detected conservative path inconsistent by Max-RPCEn1. If we say that a value $c \in D_k$ *supports* the pair of values $((i, a), (j, b))$ if it is compatible with both (i, a) and (j, b) and if it is such that $(c \geq M_{ika}) \wedge (c \geq M_{jkb}) \wedge (a \geq M_{kic}) \wedge (b \geq M_{kjc})$, then *IsValidPathConsistent* (i, a, j, b, k, c) looks for the smallest value $c \in D_k$ supporting $((i, a), (j, b))$. If at the call of this function the parameter c is not *nil*, c is the value of D_k that was supporting $((i, a), (j, b))$, and so, no value lower than c in D_k supports $((i, a), (j, b))$. This allows to never check a value of D_k twice to know whether $((i, a), (j, b))$ is valid path consistent w.r.t. k . If $M_{ika} = M_{jkb}$ and $M_{ika} \in D_k$, *IsValidPathConsistent* infers that M_{ika} supports $((i, a), (j, b))$. We could infer a support of $((i, a), (j, b))$ in D_k looking for a value in $(S_{ika} \cap S_{jkb} \cap D_k)$ but this is not cost effective.

If for all the 3-clique $\{i, j, k\}$ a value $c \in D_k$ supporting $((i, a), (j, b))$ has been found, a is added in S_{jib} to store that b is the current valid path consistent support of (i, a) in D_j , and $((i, a), (j, b))$ is added in S_{kc}^{PC} to store that c is currently supporting $((i, a), (j, b))$. Obviously, if (i, a) has no valid path consistent support in D_j , (i, a) is deleted and Max-RPCEn1 adds this value in *DeletionList* to propag its deletion. *PropagDeletion* propagates the deletion of the values of *DeletionList*. For each value (j, b) in *DeletionList*, we have to delete all the values supported by (j, b) (the values of S_{jb}) that no longer have any valid path consistent support in D_j . Furthermore, for all the pairs of values $((i, a), (k, c))$ in S_{jb}^{PC} (the pairs of values that were supported by (j, b)), *PropagDeletion* has to check whether $((i, a), (k, c))$ is still valid path consistent w.r.t. j . If no supporting value can be found in D_j , (k, c) is no longer a valid path consistent support for (i, a) and *PropagDeletion* has to look for another valid path consistent support for (i, a) in D_k to know whether (i, a) has to be deleted.

4.3 Complexity

To check whether a value a is in S_{kic} (line 7 of *PropagDeletion*) and to remove this value from S_{kic} (line 12) in constant time, we use in our implementation an array called *SupportedBy* such that *SupportedBy* $_{ika}$ is the current valid path consistent support of (i, a) in D_k , and an array *PtSupportedBy* s.t. *PtSupportedBy* $_{ika}$ points at the value a in S_{kic} , where c is the current valid

path consistent support of (i, a) in D_k . The space required by these data structures is in $O(ed)$ and so, they do not change the worst case space complexity of Max-RPCEn1 (see below).

The cost of the initialization of the data structures S , S^{PC} , M , and *InitList* (lines 1 to 5 of Max-RPCEn1) is in $O(ed)$. The time required to determine the 3-cliques of the constraint graph (lines 6 to 10) is in $O(en)$. Since *HasAValidPCSupport* removes from S_{ija} the values that are no longer in D_j , the test of line 3 is performed at most $O(d)$ times for each arc-value pair. Furthermore, the value of M_{ija} (which never decreases) is increased at each step of the second loop (lines 6 to 18) of *HasAValidPCSupport* and if M_{ija} has reached the last value of D_j , M_{ija} is set to ∞ at line 18 to stop the loop. Therefore, for each arc-value pair $[(i, j), a]$, and each value $b \in D_j$, *HasAValidPCSupport* checks at most once whether (j, b) is a valid path consistent support for (i, a) . So, Max-RPCEn1 checks the valid path consistency of at most $O(ed^2)$ pairs of values. To know whether a pair of value $((i, a), (j, b))$ is valid path consistent, for each 3-clique $\{i, j, k\}$ in the constraint graph, Max-RPCEn1 calls *IsValidPathConsistent* to look for the smallest value $c \in D_k$ supporting $((i, a), (j, b))$. When *IsValidPathConsistent* checks the valid path consistency of $((i, a), (j, b))$ w.r.t. k , it checks only the values of D_k greater than the previous support of $((i, a), (j, b))$ in D_k . So, for each support (j, b) of a value (i, a) , and each 3-clique $\{i, j, k\}$, a value of D_k is checked at most once to know whether (j, b) is a valid path consistent support for (i, a) w.r.t. k . Therefore, the complexity due to the calls to *IsValidPathConsistent* is $O(cd^3)$ where c is the number of 3-cliques in the constraint graph, and the worst case time complexity of Max-RPCEn1 is $O(en + ed^2 + cd^3)$.

The size of *InitList* is $O(ed)$ since each arc-value pair is put in this list once. When a value is deleted, it is put in *DeletionList* and it will not be put in this list any more. So, the worst case space complexity of *DeletionList* is $O(nd)$. A value a is in S_{jib} if (j, b) is the current valid path consistent support of (i, a) . Since Max-RPCEn1 looks for only one valid path consistent support for each value on each constraint, the size of the data structure S is in $O(ed)$. The data structure M is an array of $O(ed)$ counters. If a pair $((i, a), (j, b))$ is in S_{kc}^{PC} , b is the current valid path consistent support of (i, a) in D_j , $\{i, j, k\}$ is a 3-clique, and (k, c) is currently supporting $((i, a), (j, b))$. The worst case space complexity of the data structure S^{PC} is $O(cd)$ since for each value $(i, a) \in D$ and each 3-clique $\{i, j, k\}$ there is only one value $c \in D_k$ such that a pair $((i, a), (j, *))$ is in S_{kc}^{PC} and there is only one such a pair of values in S_{kc}^{PC} . Therefore, the worst case space complexity of Max-RPCEn1 is $O(ed + cd)$.

Consequently, Max-RPCEn1 has the same worst case time and space complexities as Max-RPC1.

5 Pruning efficiency

5.1 Qualitative study

We call Max-restricted path consistency enhanced (Max-RPCEn) the local consistency achieved by Max-RPCEn1. Like directional arc consistency [9], which depends on the variable ordering used, Max-RPCEn depends on the orderings used to handle the domains, *InitList*, and *DeletionList*. In the following experiments, the arc-value pairs of *InitList* are checked in the lexicographic order and *DeletionList* is a “Last In, First Out” list. To compare the pruning efficiency of Max-RPCEn to the one of the other practicable local consistencies, we use the transitive relation “stronger” introduced in [6]. A local consistency LC is *stronger* than another local consistency LC' if in any CN in which LC holds, LC' holds too. Consequently, if LC is stronger than LC' , any algorithm achieving LC deletes at least all the values removed by an algorithm achieving LC' . A local consistency LC is *strictly stronger* than another local consistency LC' if LC is stronger than LC' and there is at least one CN in which LC' holds and LC does not. In this section we compare the pruning efficiency of Max-RPCEn and conservative path consistency to the one of AC, RPC, k -RPC, Max-RPC, path inverse consistency (PIC, [12]), neighborhood inverse consistency (NIC, [12]), singleton arc consistency (SAC, [6]), singleton restricted path consistency (SRPC, [6]), and strong path consistency (strong path consistency [5,18] can be used only on very small CNs but we compare it with the practicable local consistencies because it has been widely studied).

Theorem 1. *Max-restricted path consistency enhanced is strictly stronger than Max-RPC.*

Proof. Let (i, a) be any Max-restricted path inconsistent value in a CN P . There is a constraint C_{ij} on which (i, a) has no path consistent support. Since to be a valid path consistent support for (i, a) , a value (j, b) has to be a path consistent support, (i, a) has no valid path consistent support on C_{ij} and any Max-RPCEn algorithm deletes this max-restricted path inconsistent value. So, Max-RPCEn is stronger than Max-RPC. Furthermore, Fig. 5e shows a Max-restricted path consistent CN on which Max-RPCEn does not hold (if Max-RPCEn1 uses the lexicographic order to handle *InitList*). Max-RPCEn1 looks for a valid path consistent support for $(1, a)$ in D_2 . $(2, a)$ does not support $(1, a)$. $(2, b)$ is a support for $(1, a)$ but $((1, a), (2, b))$ is not path consistent w.r.t. 4. Then, Max-RPCEn1 finds that $(2, c)$ is a valid path consistent support for $(1, a)$ and sets M_{12a} to c . After that Max-RPCEn1 has found a valid path consistent support for $(1, a)$ in D_3 and another in D_4 , it finds a support for $(1, b), (2, a), (2, b)$, and $(2, c)$ on all the constraints. Then, it checks whether $(3, a)$ has to be deleted. Max-RPCEn1 checks whether $(1, a)$ is a valid path consistent support for $(3, a)$. $(1, a)$ is compatible with $(3, a)$ but when Max-RPCEn1 checks whether $((1, a), (3, a))$ is valid path consistent w.r.t. 2, it starts its search at c ($\text{Max}(M_{12a}, M_{32a})$). Therefore, Max-RPCEn1 finds that $(1, a)$ is not a valid

path consistent support for $(3, a)$ and since $(1, b)$ is not compatible with $(3, a)$, Max-RPCEn1 deletes $(3, a)$. Consequently, Max-RPCEn is stronger than Max-RPC and since Fig. 5e shows a CN on which Max-RPC holds and Max-RPCEn does not, Max-RPCEn is strictly stronger than Max-RPC. \square

Theorem 2. *Conservative path consistency is strictly stronger than Max-restricted path consistency enhanced.*

Proof. Let (i, a) be any value removed by a Max-RPCEn algorithm. There is a constraint C_{ij} on which (i, a) has no valid path consistent support. If (i, a) does not have any support in D_j , (i, a) is arc inconsistent and any CPC algorithm deletes it. Otherwise, for any support (j, b) of (i, a) , there is a variable k linked to both i and j such that for any $c \in D_k$, $((i, a), (k, c))$ or $((j, b), (k, c))$ is conservative path inconsistent. Therefore, whatever is the support (j, b) of (i, a) , $((i, a), (j, b))$ is conservative path inconsistent. So, a CPC algorithm removes all the pairs of values $((i, a), (j, b))$ such that (j, b) is a support of (i, a) on C_{ij} . This makes arc inconsistent (i, a) and a CPC algorithm removes it. So, a CPC algorithm removes all the values deleted by a Max-RPCEn algorithm and CPC is stronger than Max-RPCEn. Furthermore, Fig. 5e shows a conservative path inconsistent CN on which Max-RPCEn1 does not delete any value if the arc-value pair $[(1, 2), a]$ is after $[(3, 1), a]$ in the ordering used to handle *InitList*. The pair of values $((1, a), (2, b))$ is not conservative path consistent and its deletion leads to the conservative path inconsistency of $((1, a), (3, a))$. So, a CPC algorithm deletes this two pairs of values and $(3, a)$ since the deletion of $((1, a), (3, a))$ leads to the arc inconsistency of $(3, a)$. If we consider an ordering on the arc-value pairs of *InitList* such that $[(1, 2), a]$ is after $[(3, 1), a]$, then Max-RPCEn1 does not delete $(3, a)$. Indeed, with such an ordering, when Max-RPCEn1 looks for a valid path consistent support for $(3, a)$ in D_1 , it has not yet found the conservative path inconsistency of $((2, b), (1, a))$ and it finds that $(1, a)$ is a valid path consistent support for $(3, a)$. So, on the CN of Fig. 5e, Max-RPCEn1 does not delete the conservative path inconsistent value $(3, a)$, and since CPC is stronger than Max-RPCEn, CPC is strictly stronger than Max-RPCEn. \square

Theorem 3. *Neighborhood inverse consistency is not stronger than Max-restricted path consistency enhanced.*

Proof. Fig. 5c shows a neighborhood inverse consistent CN on which Max-RPCEn does not hold. All the values of this CN are neighborhood inverse consistent. However, Max-RPCEn1 deletes $(2, b)$ if the lexicographic order is used to handle *InitList*. With this ordering, Max-RPCEn1 finds that $(2, a)$ is a valid path consistent support for $(1, a)$ in D_2 and then, it looks for a valid path consistent support for $(1, a)$ in D_3 . $(3, a)$ is compatible with $(1, a)$ but $((1, a), (3, a))$ is not path consistent. So, Max-RPCEn1 checks $(3, b)$. It finds that it is a valid path consistent support for $(1, a)$ and set M_{13a} to b . Then,

Max-RPCEn1 finds that $(4, a)$ is a valid path consistent support for $(1, a)$ and it proves the Max-restricted path consistency enhanced of $(1, b)$ and $(2, a)$. When Max-RPCEn1 checks whether $(1, a)$ is a valid path consistent support for $(2, b)$, it checks the valid path consistency of $((1, a), (2, b))$ w.r.t. 3 starting at b ($\max(M_{13a}, M_{23b})$). So, it finds that $(1, a)$ is not a valid path consistent support for $(2, b)$. Since $(1, b)$ is not a valid path consistent support for $(2, b)$ too, Max-RPCEn1 deletes the neighborhood inverse consistent value $(2, b)$. \square

Theorem 4. *Neighborhood inverse consistency is not stronger than conservative path consistency.*

Proof. Conservative path consistency is strictly stronger than Max-RPCEn and the stronger relation is transitive. So, if we assume that NIC is stronger than CPC, we can conclude that NIC is stronger than Max-RPCEn, which contradicts Theorem 3. \square

Theorem 5. *Conservative path consistency is not stronger than neighborhood inverse consistency.*

Proof. The CN of Fig. 5d is conservative path consistent. However, the value $(1, a)$ of this CN is neighborhood inverse inconsistent. \square

Theorem 6. *Max-restricted path consistency enhanced is not stronger than neighborhood inverse consistency.*

Proof. Conservative path consistency is strictly stronger than Max-RPCEn and the stronger relation is transitive. So, if we assume that Max-RPCEn is stronger than NIC, we can conclude that NIC is stronger than conservative path consistency, which contradicts Theorem 4. \square

Theorem 7. *Singleton arc consistency is not stronger than Max-restricted path consistency enhanced.*

Proof. Fig. 5b shows a CN that is singleton arc consistent. However, if the lexicographic order is used to handle *InitList*, Max-RPCEn1 deletes $(2, b)$. Max-RPCEn1 finds a valid path consistent support for $(1, a)$ on all the constraints, and it finds that $(2, a)$ is a valid path consistent support for $(1, b)$. When Max-RPCEn1 looks for a valid path consistent support for $(1, b)$ in D_3 , it finds the path inconsistency of $((1, b), (3, a))$ and that $(3, b)$ is not compatible with $(1, b)$. Then, it finds that $(3, c)$ is a valid path consistent support for $(1, b)$ and sets M_{13b} to c . After it has proved the max-restricted path consistency enhanced of $(1, b)$ w.r.t. C_{14} and the one of $(1, c)$ w.r.t. C_{12} , Max-RPCEn1 looks for a valid path consistent support for $(1, c)$ in D_3 . It finds that $(3, a)$ is not compatible and that $((1, c), (3, b))$ is not path consistent. Then, it finds that $(3, c)$ is a

valid path consistent support for $(1, c)$ and sets M_{13c} to c . Max-RPCEn1 finds a valid path consistent support for $(1, c)$ in D_4 and proves the max-restricted path consistency enhanced of $(2, a)$. Then Max-RPCEn1 looks for a valid path consistent support for $(2, b)$ in D_1 . $(1, a)$ is not compatible with $(2, b)$. When Max-RPCEn1 checks the valid path consistency of $((1, b), (2, b))$ w.r.t. 3, it starts its search at c ($\max(M_{13b}, M_{23b})$) and since $(3, c)$ is not compatible with $(2, b)$, Max-RPCEn1 finds that $(1, b)$ is not a valid path consistent support for $(1, b)$. Similarly, when Max-RPCEn1 checks the valid path consistency of $((1, c), (2, b))$ w.r.t. 3, it starts its search at c ($\max(M_{13c}, M_{23b})$) and since $(3, c)$ is not compatible with $(2, b)$, Max-RPCEn1 finds that $(1, c)$ is not a valid path consistent support for $(2, b)$. Therefore, $(2, b)$ does not have any valid path consistent support in D_1 and Max-RPCEn1 deletes it. \square

Theorem 8. *Singleton arc consistency is not stronger than conservative path consistency.*

Proof. Conservative path consistency is strictly stronger than Max-RPCEn and the stronger relation is transitive. So, if we assume that SAC is stronger than CPC, we can conclude that SAC is stronger than Max-RPCEn, which contradicts Theorem 7. \square

Theorem 9. *Conservative path consistency is not stronger than singleton arc consistency.*

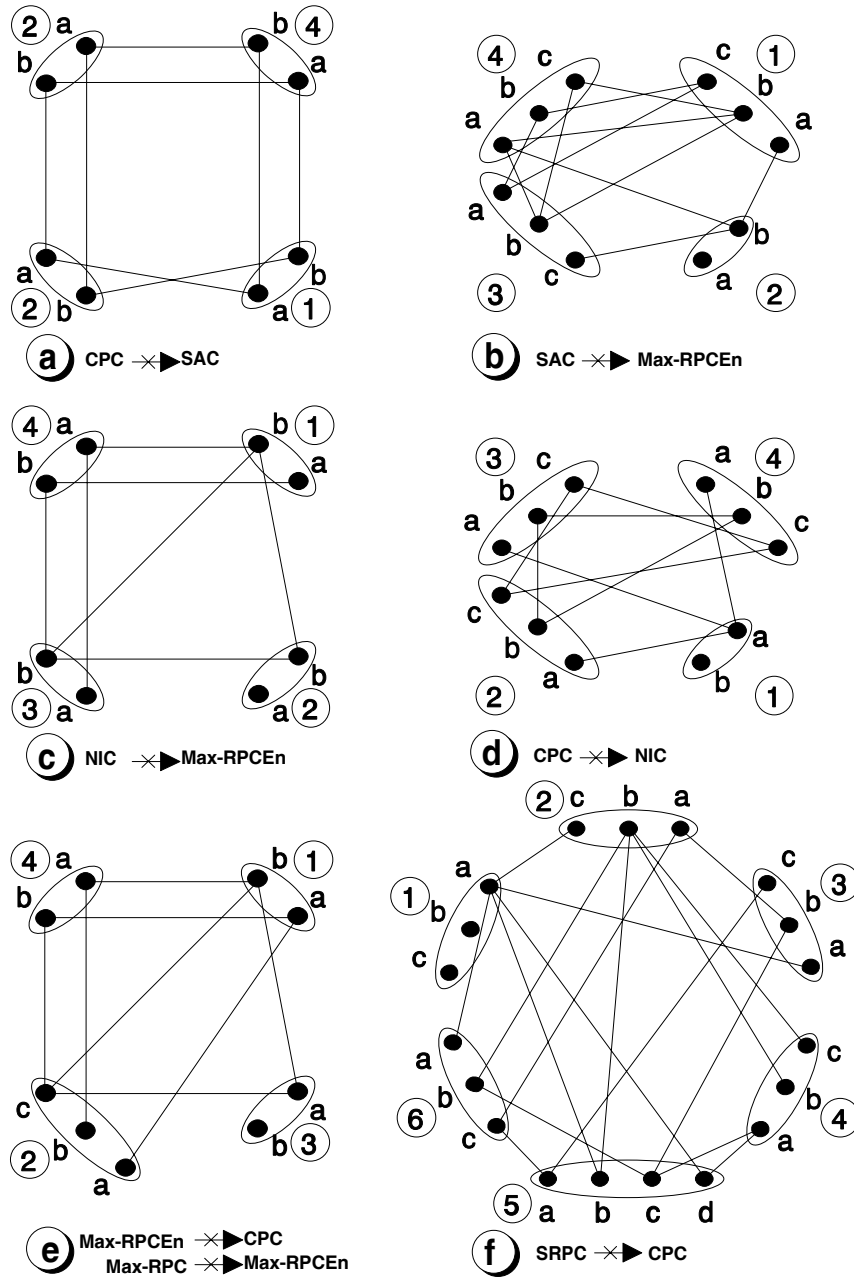
Proof. A SAC algorithm deletes all the values of the conservative path consistent CN of Fig. 5a. \square

Theorem 10. *Max-restricted path consistency enhanced is not stronger than singleton arc consistency.*

Proof. Conservative path consistency is strictly stronger than Max-RPCEn and the stronger relation is transitive. So, if we assume that Max-RPCEn is stronger than SAC, we can conclude that CPC is stronger than SAC, which contradicts Theorem 9. \square

Theorem 11. *Conservative path consistency is not stronger than singleton restricted path consistency.*

Proof. Singleton restricted path consistency is strictly stronger than singleton arc consistency [6] and the stronger relation is transitive. So, if we assume that conservative path consistency is stronger than singleton restricted path consistency, we can conclude that CPC is stronger than SAC, which contradicts Theorem 9. \square



i $\bullet \dots \bullet$ The domain of the variable i .
 $a \bullet - \bullet b$ a Forbidden pair of values.
 $A \not\rightarrow B$ B removes more values than A on this constraint network.

Fig 5: Some CNs proving the “not stronger” relations between the new local consistencies and the other practicable local consistencies.

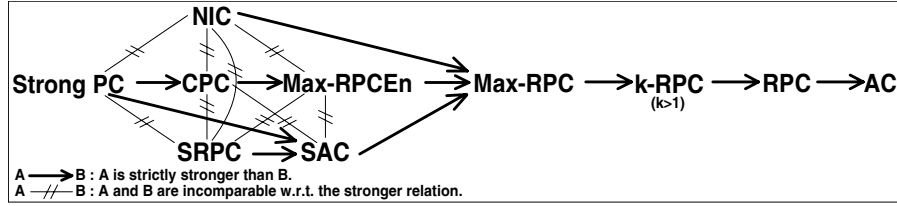


Fig. 6: Relations between the mentioned local consistencies.

Theorem 12. *Singleton restricted path consistency is not stronger than conservative path consistency.*

Proof. A CPC algorithm deletes the value $(1, a)$ of the singleton restricted path consistent CN of Fig. 5f. Indeed, a CPC algorithm deletes the pairs of values $((2, b), (5, d)), ((2, b), (5, c)), ((2, b), (6, c)), ((1, a), (2, b)), ((1, a), (6, c)), ((1, a), (5, c)), ((1, a), (3, b)), ((1, a), (5, a))$, and these deletions lead to the arc inconsistency of $(1, a)$ w.r.t. C_{15} . \square

Fig. 6 summarizes the relations between the mentioned local consistencies. There is an arrow from LC to LC' iff LC is strictly stronger than LC' . A crossed line between two local consistencies means that they are not comparable w.r.t. the stronger relation. A proof of these relations can be found in [6, 7], and if a local consistency LC is not stronger than another local consistency LC' (LC' is strictly stronger than LC , or LC and LC' are not comparable), a CN in which LC holds and LC' does not, can be found in [6, 7].

The stronger relation does not induce a total ordering. Especially, CPC and Max-RPCEn are not comparable to SAC, NIC and SRPC w.r.t. the stronger relation. However, a local consistency LC can remove all the values deleted by another local consistency LC' on most of the CNs, even though it is incomparable with LC' because of some particular CNs. Furthermore, Fig. 6 does not show if a local consistency is far more pruningful than another or if it performs only few additional value deletions.

5.2 Experimental evaluation

The experimental evaluation of this section shows how pruningful a local consistency is on random CNs with a fixed number of variables and values, when the number of constraints and the constraint tightness are changing. We used the random uniform CN generator of [13]. It involves four parameters: n the number of variables, d the common size of the initial domains, $p1$ the proportion of constraints in the network (the density $p1=1$ corresponds to the complete graph) and $p2$ the proportion of forbidden pairs of values in a constraint (the tightness). The generated problems have 40 variables and 15 values in each domain. For each local consistency and each density $p1$, two particular values of the tightness have been determined. $T_0(p1)$ is the tightness such that the local consistency does not delete any value on 50% of the CNs generated with $p1$ for density and $T_0(p1)$ for tightness. $T_{all}(p1)$ is the tightness such that the local

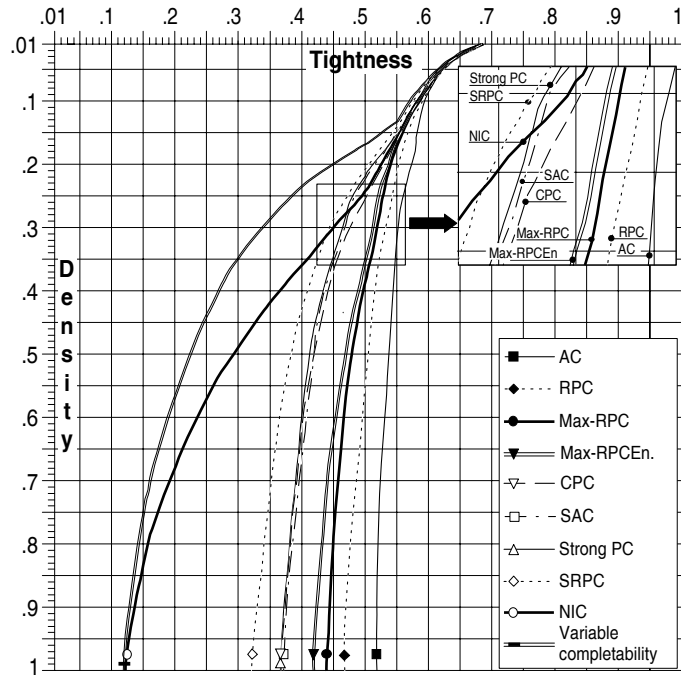


Fig. 7: The T_0 bounds for random CNs with $n=40$ and $d=15$.

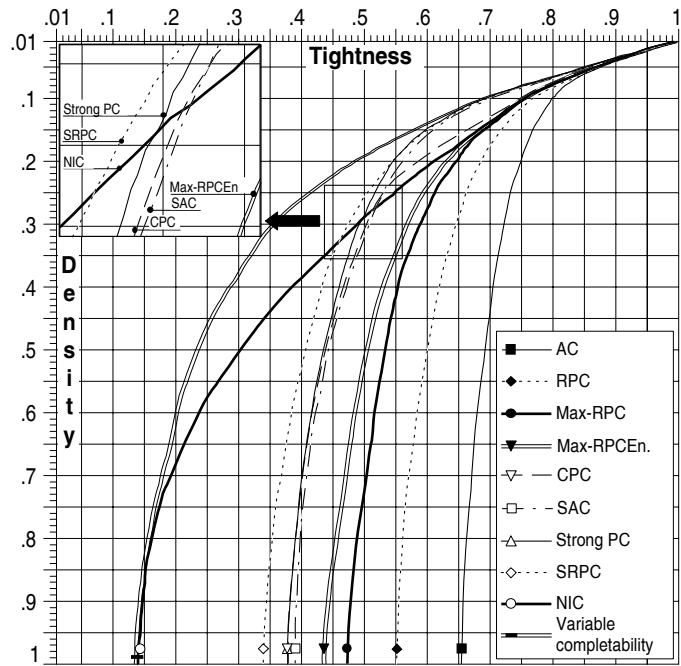


Fig. 8: The T_{all} bounds for random CNs with $n=40$ and $d=15$.

consistency finds the inconsistency of 50% of the CNs generated with tightness $T_{all}(p1)$ and density $p1$. For all the mentioned local consistencies, the values $T_0(p1)$ and $T_{all}(p1)$ for any density $p1$ are given in Fig. 7 and Fig. 8 respectively. We also show these bounds for the variable completability filtering [11], which removes the values that do not belong to any solution, and thus is the strongest filtering we can have when we limit filtering to the domains. Many instances have to be considered to determine the T_0 and T_{all} bounds. This explains that the generated problems are relatively small.

On sparse random uniform CNs, conservative path consistency is less pruneful than singleton arc consistency, but on more dense CNs, CPC has a better pruning efficiency than SAC. These two local consistencies removes almost all the strong path inconsistent values. Obviously, on complete CNs, CPC is strong path consistency. However, even on relatively sparse CNs, the T_0 and T_{all} bounds of CPC and strong PC are very close. Compared to Max-RPC, Max-RPCen is a substantial enhancement w.r.t. the pruning efficiency.

6 Time efficiency

The same random uniform CN generator is used to compare the time efficiency. Fig. 9 shows the results on relatively sparse CNs having 1000 variables and 20 values in each initial domain, and Fig. 10 presents performances on complete CNs with $n=100$ and $d=30$. For each tightness, 50 instances were generated, and Fig. 9 and Fig. 10 show mean values obtained on a Pentium II-266 Mhz with 64 Mo of memory under Linux.

The advantage of using Max-RPCen1 on sparse CNs is obvious. The bidirectionality allows substantial constraint check savings and although Max-RPCen1 deletes more values than Max-RPC1, it requires less cpu time. There is “few” 3-cliques in the constraint graph of these sparse CNs, and Max-RPC1 and Max-RPCen1 requires at most 108 seconds (On all the CNs generated with a tightness lower than 0.69, PC8, the PC algorithm presented in [5], requires more than 68 hours).

On dense CNs, Max-RPCen1 still requires less constraint checks and list checks than Max-RPC1. However, there is many 3-cliques in the constraint graph, and Max-RPCen1 detects the conservative path inconsistency of many pairs of values. Therefore, for each value (i, a) and each constraint C_{ij} , Max-RPCen1 checks more values of D_j to find a valid path consistent support for (i, a) than Max-RPC1 to find a path consistent support. So, Max-RPCen1 requires more cpu time than Max-RPC1 as long as it removes only few values. However, the cpu time performances of Max-RPCen1 and Max-RPC1 remain of the same order of magnitude, and the improvement of the pruning efficiency is significant on these complete CNs.

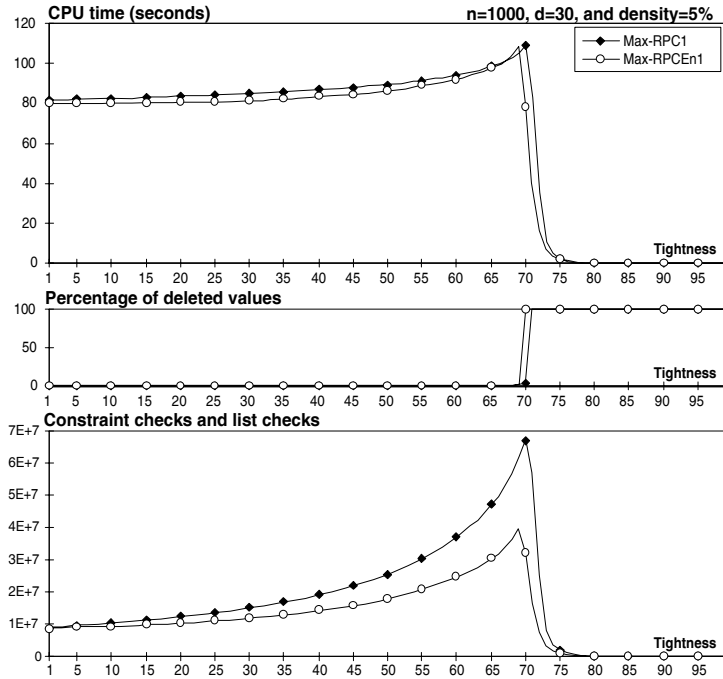


Fig. 9: Experimental evaluation on CNs with $n=1000$, $d=30$, and density=0.05.

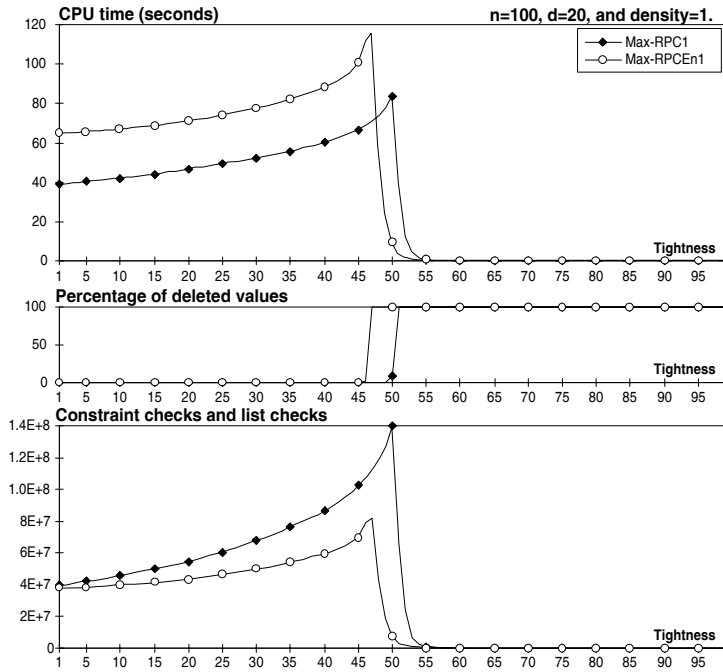


Fig. 10: Experimental evaluation on complete CNs with $n=100$, and $d=20$.

7 Conclusion

In this paper, we proposed two new local consistencies: conservative path consistency and Max-restricted path consistency enhanced. A study of their pruning efficiency with the most practicable local consistencies showed that conservative path consistency deletes less values than strong path consistency only on very sparse constraint networks and that Max-RPCEn removes far more values than Max-RPC. In addition to the pruning enhancement, Max-RPCEn1, the algorithm presented to achieve Max-restricted path consistency enhanced, has cpu time performances that remain comparable to those of Max-RPC1, and it requires less constraint checks. It is therefore one of the most worthwhile filtering algorithms.

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