5.11 alldifferent_modulo

DESCRIPTION LINKS GRAPH AUTOMATON

Origin Derived from alldifferent.

Constraint alldifferent_modulo(VARIABLES, M)

Synonyms alldiff_modulo, alldistinct_modulo.

Arguments

VARIABLES	:	collection(var−dvar)
M	:	int

Restrictions

required(VARIABLES, var)
M > 0
M ≥ |VARIABLES|

Purpose Enforce all variables of the collection VARIABLES to have a distinct rest when divided by M.

Example $(\langle 25, 1, 14, 3\rangle, 5)$

The equivalence classes associated with values 25, 1, 14 and 3 are respectively equal to $25 \bmod 5 = 0$, $1 \bmod 5 = 1$, $14 \bmod 5 = 4$ and $3 \bmod 5 = 3$. Since they are distinct the alldifferent_modulo constraint holds.

Typical

|VARIABLES| > 2
M > 1

Symmetries

- Items of VARIABLES are permutable.
- A value u of VARIABLES.var can be renamed to any value v such that v is congruent to u modulo M.
- Two distinct values u and v of VARIABLES.var such that $u \bmod \mathtt{M} \neq v \bmod \mathtt{M}$ can be swapped.

See also **specialisation:** alldifferent *(*variable mod constant *replaced by* variable*)*.

Keywords **characteristic of a constraint:** modulo, all different, automaton, automaton with array of counters.

constraint type: value constraint.

filtering: arc-consistency.

final graph structure: one_succ.

Arc input(s)	VARIABLES
Arc generator	$CLIQUE \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var $\bmod$ M $=$ variables2.var $\bmod$ M
Graph property(ies)	**MAX_NSCC** ≤ 1
Graph class	ONE_SUCC

Graph model Exploit the same model used for the alldifferent constraint. We replace the binary *equality* constraint by another equivalence relation depicted by the arc constraint. We generate a *clique* with a binary *equality modulo* M constraint between each pair of vertices (including a vertex and itself) and state that the size of the largest strongly connected component should not exceed 1.

Parts (A) and (B) of Figure 5.14 respectively show the initial and final graph associated with the **Example** slot. Since we use the **MAX_NSCC** graph property we show one of the largest strongly connected component of the final graph.

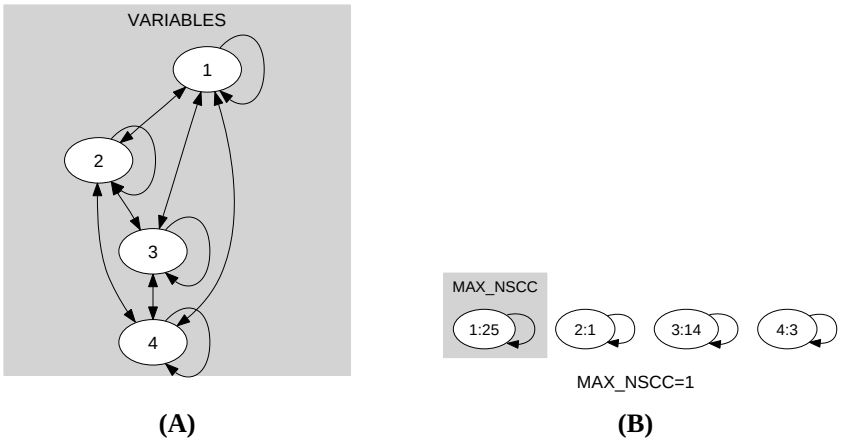

Figure 5.14: Initial and final graph of the alldifferent_modulo constraint

Automaton

Figure 5.15 depicts the automaton associated with the `alldifferent_modulo` constraint. To each item of the collection `VARIABLES` corresponds a signature variable $\mathtt{S}_i$ that is equal to 1. The automaton counts for each equivalence class the number of used values and finally imposes that each equivalence class is used at most one time.

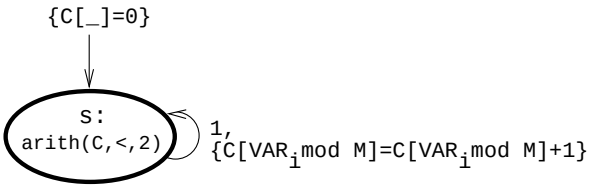

Figure 5.15: Automaton of the `alldifferent_modulo` constraint