

### 5.30 atleast\_nvalue

	DESCRIPTION	LINKS	GRAPH
<b>Origin</b>	[308]		
<b>Constraint</b>	atleast_nvalue(NVAL, VARIABLES)		
<b>Synonym</b>	k_diff.		
<b>Arguments</b>	NVAL : dvar VARIABLES : collection(var-dvar)		
<b>Restrictions</b>	$NVAL \geq 0$ $NVAL \leq  VARIABLES $ required(VARIABLES, var)		
<b>Purpose</b>	The number of distinct values taken by the variables of the collection VARIABLES is greater than or equal to NVAL.		
<b>Example</b>	$(2, \langle 3, 1, 7, 1, 6 \rangle)$ <p>The atleast_nvalue constraint holds since the collection <math>\langle 3, 1, 7, 1, 6 \rangle</math> involves at least 2 distinct values (i.e., in fact 4 distinct values).</p>		
<b>Typical</b>	$NVAL > 0$ $NVAL <  VARIABLES $ $ VARIABLES  > 1$		
<b>Symmetries</b>	<ul style="list-style-type: none"> <li>• NVAL can be decreased to any value <math>\geq 0</math>.</li> <li>• Items of VARIABLES are <i>permutable</i>.</li> <li>• All occurrences of two distinct values of VARIABLES.var can be <i>swapped</i>; all occurrences of a value of VARIABLES.var can be <i>renamed</i> to any unused value.</li> </ul>		
<b>Remark</b>	The atleast_nvalue constraint was first introduced by J.-C. Régin under the name k_diff in [308]. Later on the atleast_nvalue constraint was introduced together with the atleast_nvalue constraint by C. Bessière <i>et al.</i> in a article [56] providing filtering algorithms for the nvalue constraint.		
<b>Algorithm</b>	[56] provides a sketch of a filtering algorithm enforcing <i>arc-consistency</i> for the atleast_nvalue constraint. This algorithm is based on the maximal matching in a bi-partite graph.		
<b>See also</b>	<b>comparison swapped:</b> atmost_nvalue. <b>implied by:</b> nvalue ( $\geq NVAL$ replaced by $= NVAL$ ). <b>uses in its reformulation:</b> not_all_equal.		

**Keywords**

**constraint type:** counting constraint, value partitioning constraint.

**filtering:** bipartite matching, arc-consistency.

**final graph structure:** strongly connected component, equivalence.

**modelling:** number of distinct equivalence classes, number of distinct values.

<b>Arc input(s)</b>	VARIABLES
<b>Arc generator</b>	<i>CLIQUE</i> $\mapsto$ collection(variables1, variables2)
<b>Arc arity</b>	2
<b>Arc constraint(s)</b>	variables1.var = variables2.var
<b>Graph property(ies)</b>	$\text{NSCC} \geq \text{NVAL}$
<b>Graph class</b>	<i>EQUIVALENCE</i>

**Graph model**

Parts (A) and (B) of Figure 5.58 respectively show the initial and final graph associated with the **Example** slot. Since we use the  $\text{NSCC}$  graph property we show the different strongly connected components of the final graph. Each strongly connected component corresponds to a specific value that is assigned to some variables of the VARIABLES collection. The 4 following values 1, 3, 6 and 7 are used by the variables of the VARIABLES collection.

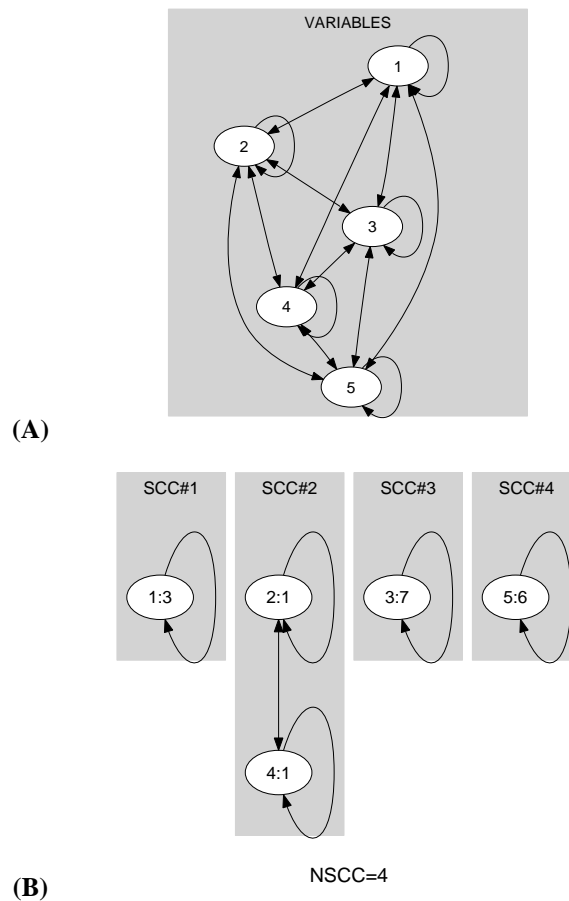


Figure 5.58: Initial and final graph of the `atleast_nvalue` constraint

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