

5.163 indexed_sum

	DESCRIPTION	LINKS	GRAPH
Origin	N. Beldiceanu		
Constraint	indexed_sum(ITEMS, TABLE)		
Arguments	ITEMS : <code>collection(index-dvar, weight-dvar)</code> TABLE : <code>collection(index-int, summation-dvar)</code>		
Restrictions	$ ITEMS > 0$ $ TABLE > 0$ <code>required(ITEMS, [index, weight])</code> $ITEMS.index \geq 1$ $ITEMS.index \leq TABLE $ <code>required(TABLE, [index, summation])</code> $TABLE.index \geq 1$ $TABLE.index \leq TABLE $ <code>increasing_seq(TABLE, index)</code>		
Purpose	<p>Given several items of the collection ITEMS (each of them having a specific fixed index as well as a weight that may be negative or positive), and a table TABLE (each entry of TABLE corresponding to a summation variable), assign each item to an entry of TABLE so that the sum of the weights of the items assigned to that entry is equal to the corresponding summation variable.</p>		
Example	$\left(\begin{array}{l} \left\langle \begin{array}{ll} index - 3 & weight - -4, \\ index - 1 & weight - 6, \\ index - 3 & weight - 1 \end{array} \right\rangle, \\ \left\langle \begin{array}{ll} index - 1 & summation - 6, \\ index - 2 & summation - 0, \\ index - 3 & summation - -3 \end{array} \right\rangle \end{array} \right)$		
	<p>The <code>indexed_sum</code> constraint holds since the summation variables associated with each entry of TABLE are equal to the sum of the weights of the items assigned to the corresponding entry:</p> <ul style="list-style-type: none"> • $TABLE[1].summation = ITEMS[2].weight = 6$ (since $TABLE[1].index = ITEMS[2].index = 1$), • $TABLE[2].summation = 0$ (since $TABLE[2].index = 2$ does not occur as a value of the index attribute of an item of ITEMS), • $TABLE[3].summation = ITEMS[1].weight + ITEMS[3].weight = -4 + 1 = -3$ (since $TABLE[3].index = ITEMS[1].index = ITEMS[3].index = 3$). 		
Typical	$ ITEMS > 1$ <code>range(ITEMS.index) > 1</code> $ TABLE > 1$ <code>range(TABLE.summation) > 1</code>		

Symmetries

- Items of ITEMS are [permutable](#).
- Items of TABLE are [permutable](#).

Reformulation

The `indexed_sum`(ITEMS, TABLE) constraint can be expressed in term of a set of reified constraints and of `|TABLE|` arithmetic constraints (i.e., [scalar_product](#) constraints).

1. For each item `ITEMS[i]` ($i \in [1, |\text{ITEMS}|]$) and for each table entry j ($j \in [1, |\text{TABLE}|]$) of TABLE we create a 0-1 variable B_{ij} that will be set to 1 if and only if `ITEMS[i].index` is fixed to j (i.e., $B_{ij} \Leftrightarrow \text{ITEMS}[i].\text{index} = j$).
2. For each entry j of the table TABLE, we impose the sum `ITEMS[1].weight · B1j + ITEMS[2].weight · B2j + ... + ITEMS[|ITEMS|].weight · B|ITEMS|j` to be equal to `TABLE[j].summation`.

See also

[specialisation: bin_packing](#) (negative contribution not allowed, effective use variable for each bin replaced by an overall fixed capacity), [bin_packing_capa](#) (negative contribution not allowed, effective use variable for each bin replaced by a fixed capacity for each bin).

[used in graph description: sum_ctr](#).

Keywords

[application area: assignment](#).

[modelling: variable indexing, variable subscript](#).

	For all items of TABLE:
Arc input(s)	ITEMS TABLE
Arc generator	<code>PRODUCT</code> \mapsto <code>collection(items, table)</code>
Arc arity	2
Arc constraint(s)	<code>items.index = table.index</code>
Sets	<code>SUCC</code> \mapsto $\left[\begin{array}{l} \text{source,} \\ \text{variables} - \text{col} \left(\begin{array}{l} \text{VARIABLES} - \text{collection}(\text{var} - \text{dvar}), \\ [\text{item}(\text{var} - \text{ITEMS.weight})] \end{array} \right) \end{array} \right]$
Constraint(s) on sets	<code>sum_ctr(variables, =, TABLE.summation)</code>

Graph model

We enforce the `sum_ctr` constraint on the weight of the items that are assigned to the same entry. Within the context of the **Example** slot, part (A) of Figure 5.329 shows the initial graphs associated with entries 1, 2 and 3 (i.e., one initial graph for each item of the TABLE collection). Part (B) of Figure 5.329 shows the corresponding final graphs associated with entries 1 and 3. Each source vertex of the final graph can be interpreted as an item assigned to a specific entry of TABLE.

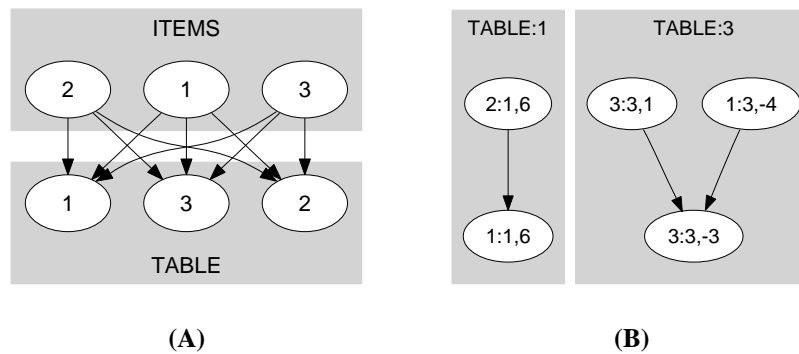


Figure 5.329: Initial and final graph of the `indexed_sum` constraint

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