5.193 lex_chain_less

DESCRIPTION LINKS GRAPH

Origin [86]

Constraint lex_chain_less(VECTORS)

Usual name lex_chain

Type VECTOR : collection(var−dvar)

Argument VECTORS : collection(vec − VECTOR)

Restrictions
required(VECTOR, var)
required(VECTORS, vec)
same_size(VECTORS, vec)

Purpose For each pair of consecutive vectors $\mathtt{VECTOR}_i$ and $\mathtt{VECTOR}_{i+1}$ of the VECTORS collection we have that $\mathtt{VECTOR}_i$ is lexicographically strictly less than $\mathtt{VECTOR}_{i+1}$. Given two vectors, $\vec{X}$ and $\vec{Y}$ of n components, $\langle X_0, \ldots, X_{n-1}\rangle$ and $\langle Y_0, \ldots, Y_{n-1}\rangle$, $\vec{X}$ is *lexicographically strictly less than* $\vec{Y}$ if and only if $X_0 < Y_0$ or $X_0 = Y_0$ and $\langle X_1, \ldots, X_{n-1}\rangle$ is lexicographically strictly less than $\langle Y_1, \ldots, Y_{n-1}\rangle$.

Example

$$\left(\left\langle\begin{array}{l}\mathtt{vec}-\langle 5,2,3,9\rangle,\\ \mathtt{vec}-\langle 5,2,6,2\rangle,\\ \mathtt{vec}-\langle 5,2,6,3\rangle\end{array}\right\rangle\right)$$

The lex_chain_less constraint holds since:

- The first vector $\langle 5, 2, 3, 9\rangle$ of the VECTORS collection is lexicographically strictly less than the second vector $\langle 5, 2, 6, 2\rangle$ of the VECTORS collection.
- The second vector $\langle 5, 2, 6, 2\rangle$ of the VECTORS collection is lexicographically strictly less than the third vector $\langle 5, 2, 6, 3\rangle$ of the VECTORS collection.

Usage This constraint was motivated for breaking symmetry: more precisely when one wants to lexicographically order the consecutive columns of a matrix of decision variables. A further motivation is that using a set of lexicographic ordering constraints between two vectors does usually not allows to come up with a complete pruning.

Algorithm A filtering algorithm achieving arc-consistency for a chain of lexicographical ordering constraints is presented in [86].

Six different ways of integrating a chain of lexicographical ordering constraints within non-overlapping constraints like diffn or geost and within their corresponding necessary condition like the cumulative constraint are shown in [2].

Systems lexChain in **Choco**, lex_chain in **SICStus**.

See also

common keyword: `geost` *(symmetry, lexicographic ordering on the origins of* `tasks`*,* `rectangles`*, ...)*, `lex_between`, `lex_greater`, `lex_greatereq`, `lex_lesseq` *(lexicographic order)*.

implied by: `strict_lex2`.

implies: `lex_alldifferent`, `lex_chain_lesseq`.

part of system of constraints: `lex_less`.

related: `cumulative`, `diffn` *(lexicographic ordering on the origins of* `tasks`*,* `rectangles`*, ...)*.

system of constraints: `strict_lex2`.

used in graph description: `lex_less`.

Keywords

application area: floor planning problem.

characteristic of a constraint: vector.

constraint type: decomposition, order constraint, system of constraints.

filtering: arc-consistency.

heuristics: heuristics and lexicographical ordering.

modelling: degree of diversity of a set of solutions.

modelling exercises: degree of diversity of a set of solutions.

symmetry: symmetry, matrix symmetry, lexicographic order.

Arc input(s)	`VECTORS`
Arc generator	*PATH* ↦ `collection(vectors1, vectors2)`
Arc arity	2
Arc constraint(s)	`lex_less(vectors1.vec, vectors2.vec)`
Graph property(ies)	$\mathbf{NARC}$ = |`VECTORS`| − 1

Graph model Parts (A) and (B) of Figure 5.375 respectively show the initial and final graph associated with the **Example** slot. Since we use the $\mathbf{NARC}$ graph property, the arcs of the final graph are stressed in bold. The `lex_chain_less` constraint holds since all the arc constraints of the initial graph are satisfied.

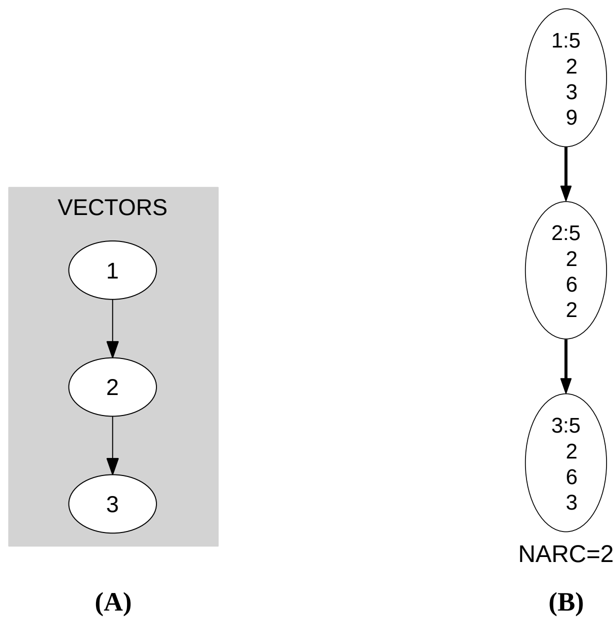

Figure 5.375: Initial and final graph of the `lex_chain_less` constraint

Signature Since we use the *PATH* arc generator on the `VECTORS` collection the number of arcs of the initial graph is equal to $|\texttt{VECTORS}| - 1$. For this reason we can rewrite $\mathbf{NARC} = |\texttt{VECTORS}| - 1$ to $\mathbf{NARC} \geq |\texttt{VECTORS}| - 1$ and simplify $\underline{\overline{\mathbf{NARC}}}$ to $\overline{\mathbf{NARC}}$.