5.282 same_interval

DESCRIPTION LINKS GRAPH

Origin Derived from same.

Constraint same_interval(VARIABLES1, VARIABLES2, SIZE_INTERVAL)

Arguments

VARIABLES1	:	collection(var−dvar)
VARIABLES2	:	collection(var−dvar)
SIZE_INTERVAL	:	int

Restrictions

|VARIABLES1| = |VARIABLES2|
required(VARIABLES1, var)
required(VARIABLES2, var)
SIZE_INTERVAL > 0

Purpose Let N_i (respectively M_i) denote the number of variables of the collection VARIABLES1 (respectively VARIABLES2) that take a value in the interval [SIZE_INTERVAL $\cdot$ i, SIZE_INTERVAL $\cdot$ i + SIZE_INTERVAL − 1]. For all integer i we have $N_i = M_i$.

Example

$$\left(\begin{array}{l} \left\langle\begin{array}{l} \texttt{var}-1, \\ \texttt{var}-7, \\ \texttt{var}-6, \\ \texttt{var}-0, \\ \texttt{var}-1, \\ \texttt{var}-7 \end{array}\right\rangle, \\ \left\langle\begin{array}{l} \texttt{var}-8, \\ \texttt{var}-8, \\ \texttt{var}-8, \\ \texttt{var}-0, \\ \texttt{var}-1, \\ \texttt{var}-2 \end{array}\right\rangle, 3 \end{array}\right)$$

In the example, the third argument SIZE_INTERVAL = 3 defines the following family of intervals $[3 \cdot k, 3 \cdot k + 2]$, where k is an integer. Consequently the values of the collection $\langle 1, 7, 6, 0, 1, 7\rangle$ are respectively located within intervals $[0, 2]$, $[6, 8]$, $[6, 8]$, $[0, 2]$, $[0, 2]$, $[6, 8]$. Therefore intervals $[0, 2]$ and $[6, 8]$ are respectively used 3 and 3 times. Similarly, the values of the collection $\langle 8, 8, 8, 0, 1, 2\rangle$ are respectively located within intervals $[6, 8]$, $[6, 8]$, $[6, 8]$, $[0, 2]$, $[0, 2]$, $[0, 2]$. As before intervals $[0, 2]$ and $[6, 8]$ are respectively used 3 and 3 times. Consequently the same_interval constraint holds. Figure 5.514 illustrates this correspondence.

Symmetries

- Arguments are permutable w.r.t. permutation (VARIABLES1, VARIABLES2).
- Items of VARIABLES1 are permutable.
- Items of VARIABLES2 are permutable.
- An occurrence of a value of VARIABLES.var that belongs to the k-th interval, of size SIZE_INTERVAL, can be replaced by any other value of the same interval.

Algorithm See algorithm of the `same` constraint.

Used in `k_same_interval`.

See also **implies:** `used_by_interval`.

soft variant: `soft_same_interval_var` *(variable-based violation measure)*.

specialisation: `same` *(*`variable`/`constant` *replaced by* `variable`*)*.

system of constraints: `k_same_interval`.

Keywords **combinatorial object:** permutation.

constraint arguments: constraint between two collections of variables.

modelling: interval.

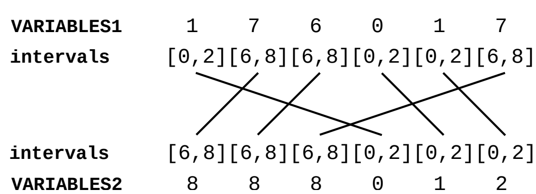


Figure 5.514: Correspondence between the intervals associated with collection $\langle 1,7,6,0,1,7\rangle$ and with collection $\langle 8,8,8,0,1,2\rangle$

Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	$PRODUCT \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var/SIZE_INTERVAL = variables2.var/SIZE_INTERVAL
Graph property(ies)	• for all connected components: **NSOURCE**=**NSINK** • **NSOURCE**= |VARIABLES1| • **NSINK**= |VARIABLES2|

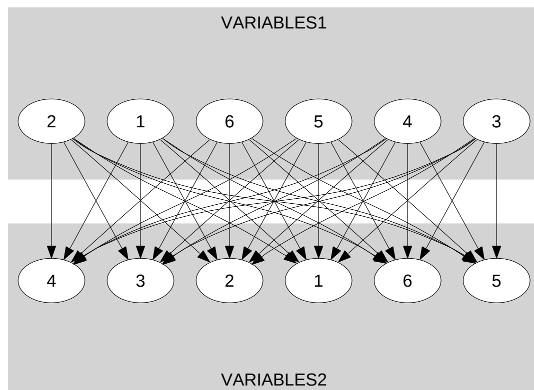
Graph model Parts (A) and (B) of Figure 5.515 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NSOURCE** and **NSINK** graph properties, the source and sink vertices of the final graph are stressed with a double circle. Since there is a constraint on each connected component of the final graph we also show the different connected components. Each of them corresponds to an equivalence class according to the arc constraint. The same_interval constraint holds since:

- Each connected component of the final graph has the same number of sources and of sinks.
- The number of sources of the final graph is equal to |VARIABLES1|.
- The number of sinks of the final graph is equal to |VARIABLES2|.

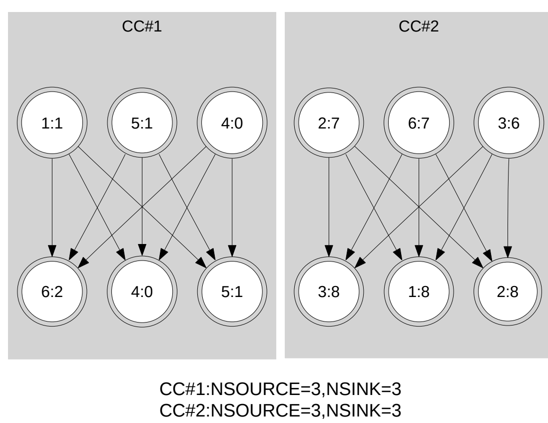
Signature Since the initial graph contains only sources and sinks, and since isolated vertices are eliminated from the final graph, we make the following observations:

- Sources of the initial graph cannot become sinks of the final graph,
- Sinks of the initial graph cannot become sources of the final graph.

From the previous observations and since we use the $PRODUCT$ arc generator on the collections VARIABLES1 and VARIABLES2, we have that the maximum number of sources and sinks of the final graph is respectively equal to |VARIABLES1| and |VARIABLES2|. Therefore we can rewrite **NSOURCE** = |VARIABLES1| to **NSOURCE** $\geq$ |VARIABLES1| and simplify $\overline{\mathbf{NSOURCE}}$ to $\overline{\mathbf{NSOURCE}}$. In a similar way, we can rewrite **NSINK** = |VARIABLES2| to **NSINK** $\geq$ |VARIABLES2| and simplify $\overline{\mathbf{NSINK}}$ to $\overline{\mathbf{NSINK}}$.



(A)



(B)

Figure 5.515: Initial and final graph of the `same_interval` constraint