5.283 same_modulo

DESCRIPTION LINKS GRAPH

Origin

Derived from same.

Constraint

same_modulo(VARIABLES1, VARIABLES2, M)

Arguments

VARIABLES1	:	collection(var−dvar)
VARIABLES2	:	collection(var−dvar)
M	:	int

Restrictions

|VARIABLES1| = |VARIABLES2|
required(VARIABLES1, var)
required(VARIABLES2, var)
M > 0

Purpose

For each integer R in $[0, \texttt{M}-1]$, let $N1_R$ (respectively $N2_R$) denote the number of variables of VARIABLES1 (respectively VARIABLES2) that have R as a rest when divided by M. For all R in $[0, \texttt{M}-1]$ we have that $N1_R = N2_R$.

Example

$$\left(\begin{array}{l}\left\langle\begin{array}{l}\texttt{var}-1,\\ \texttt{var}-9,\\ \texttt{var}-1,\\ \texttt{var}-5,\\ \texttt{var}-2,\\ \texttt{var}-1\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\texttt{var}-6,\\ \texttt{var}-4,\\ \texttt{var}-1,\\ \texttt{var}-1,\\ \texttt{var}-5,\\ \texttt{var}-5\end{array}\right\rangle, 3\end{array}\right)$$

The values of the first collection $\langle 1, 9, 1, 5, 2, 1\rangle$ are respectively associated with the equivalence classes $1 \bmod 3 = 1$, $9 \bmod 3 = 0$, $1 \bmod 3 = 1$, $5 \bmod 3 = 2$, $2 \bmod 3 = 2$, $1 \bmod 3 = 1$. Therefore the equivalence classes 0, 1, and 2 are respectively used 1, 3, and 2 times. Similarly, the values of the second collection $\langle 6, 4, 1, 1, 5, 5\rangle$ are respectively associated with the equivalence classes $6 \bmod 3 = 0$, $4 \bmod 3 = 1$, $1 \bmod 3 = 1$, $1 \bmod 3 = 1$, $5 \bmod 3 = 2$, $5 \bmod 3 = 2$. Therefore the equivalence classes 0, 1, and 2 are respectively used 1, 3, and 2 times. Consequently the same_modulo constraint holds. Figure 5.516 illustrates this correspondence.

Symmetries

- Arguments are permutable w.r.t. permutation (VARIABLES1, VARIABLES2).
- Items of VARIABLES1 are permutable.
- Items of VARIABLES2 are permutable.
- An occurrence of a value u of VARIABLES.var can be replaced by any other value v such that v is congruent to u modulo M.

Used in k_same_modulo.

See also

implies: used_by_modulo.

soft variant: soft_same_modulo_var *(variable-based violation measure)*.

specialisation: same *(*variable mod constant *replaced by* variable*)*.

system of constraints: k_same_modulo.

Keywords

characteristic of a constraint: modulo.

combinatorial object: permutation.

constraint arguments: constraint between two collections of variables.

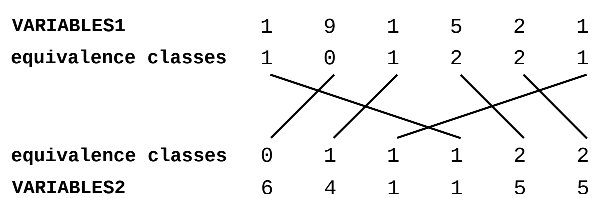


Figure 5.516: Correspondence between the equivalence classes associated with collection $\langle 1, 9, 1, 5, 2, 1\rangle$ and with collection $\langle 6, 4, 1, 1, 5, 5\rangle$

Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	$PRODUCT \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var mod M = variables2.var mod M
Graph property(ies)	• for all connected components: **NSOURCE**=**NSINK** • **NSOURCE**= |VARIABLES1| • **NSINK**= |VARIABLES2|

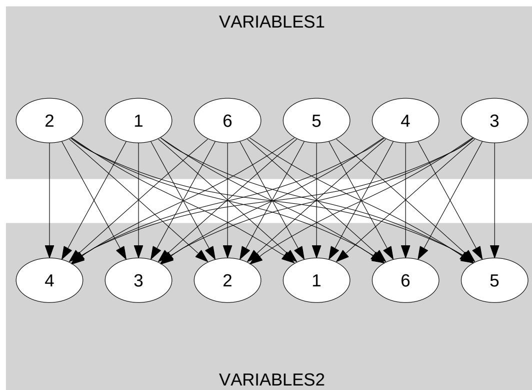
Graph model Parts (A) and (B) of Figure 5.517 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NSOURCE** and **NSINK** graph properties, the source and sink vertices of the final graph are stressed with a double circle. Since there is a constraint on each connected component of the final graph we also show the different connected components. Each of them corresponds to an equivalence class according to the arc constraint. The same_modulo constraint holds since:

- Each connected component of the final graph has the same number of sources and of sinks.
- The number of sources of the final graph is equal to |VARIABLES1|.
- The number of sinks of the final graph is equal to |VARIABLES2|.

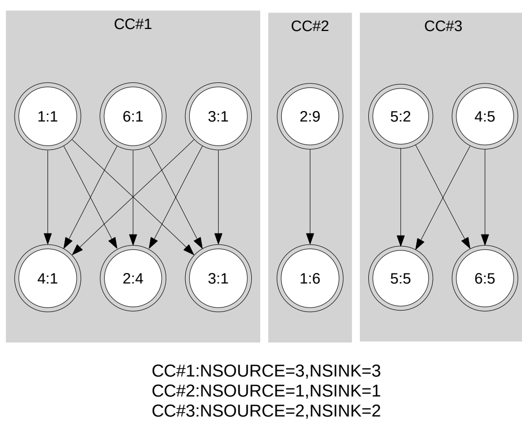
Signature Since the initial graph contains only sources and sinks, and since isolated vertices are eliminated from the final graph, we make the following observations:

- Sources of the initial graph cannot become sinks of the final graph,
- Sinks of the initial graph cannot become sources of the final graph.

From the previous observations and since we use the $PRODUCT$ arc generator on the collections VARIABLES1 and VARIABLES2, we have that the maximum number of sources and sinks of the final graph is respectively equal to |VARIABLES1| and |VARIABLES2|. Therefore we can rewrite **NSOURCE** = |VARIABLES1| to **NSOURCE** $\geq$ |VARIABLES1| and simplify $\underline{\overline{\mathbf{NSOURCE}}}$ to $\overline{\mathbf{NSOURCE}}$. In a similar way, we can rewrite **NSINK** = |VARIABLES2| to **NSINK** $\geq$ |VARIABLES2| and simplify $\underline{\overline{\mathbf{NSINK}}}$ to $\overline{\mathbf{NSINK}}$.



(A)



(B)

Figure 5.517: Initial and final graph of the `same_modulo` constraint